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REFLECTION AND TRANSMISSION COEFFICIENTS FOR PLANE WAVES IN ELASTIC MEDIA*

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ABSTRACT‡

The results are given of a systematic series of calculations on the coefficients of reflection and transmission for plane waves incident on elastic interfaces. Tables are given for the amplitudes of the reflected and transmitted longitudinal and transverse waves, for the intensities of these components, and for the fractions of the incident energy carried away by them. For incident longitudinal waves calculations were carried out for angles of incidence between 0 and 30° with 5° intervals. For incident transverse waves polarized in the plane of incidence results are given for four angles of incidence up to approximately 16°. For incident transverse waves polarized normal to the plane of incidence the calculations were carried through for all angles of incidence—in steps of 5°—up to total reflection. All the calculations were carried through for interfacial density ratios of 0.7 to 1.3 in steps of 0.1, and interfacial velocity ratios between 0.5 and 2.0 in steps of 0.25.

INTRODUCTION

Although Knott[1] derived the equations for the reflection and refraction of plane waves at interfaces in elastic media 40 years ago, there have been no complete systematic calculations carried through giving the numerical values for the coefficients. Knott himself, and several later investigators[2] made the calculations for a number of

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‡ The extensive tables of reflection-coefficient data are published in full for reference purposes and at the specific request of S.E.G. members having need for them. Perhaps a more immediately instructive presentation was contained in the original manuscript which included a complete set of curves plotted from the tabulated data. Since space limitations precluded publication of both tables and curves, the tables were selected, since these data present accurate values which may be plotted in any manner desired by the user. In lieu of the original curves and conveniently to show the nature of the results given in the tables the author prepared Fig. 3. EDITOR

[1] C. G. Knott, Phil. Mag. *48*, 64, (1899).

[2] H. Blut, Z. f. Geophysik, *8*, 130, 305, (1932); E. Wanner, Gerl. Beitr. z. Geophysik, *32*, 231, (1931); L. B. Slichter and V. G. Gabriel, loc. cit. *38*, 228, 239, (1933).

special cases which were of interest in the study of the general problem of the structure of the earth's crust. In some cases[3] the transverse waves created at the interfaces between the elastic strata upon the incidence of longitudinal waves were neglected. In others the reflecting medium was supposed to be incapable of carrying any energy, as in the case of the atmosphere overlying the earth's surface.[4] In still another investigation[5] approximations were deliberately introduced in the calculations of the reflection coefficient, although evidence was deduced that these approximations were not especially serious.

Because the whole subject of applied seismology, especially with regard to oil prospecting, depends upon the reflection and refraction of essentially plane waves at interfaces between earth strata differing in elastic properties, there evidently should be interest in the development of systematic tables for the theoretical reflection and transmission coefficients for the various types of interfaces. For with such information at hand it should be possible not only to predict the intensity of the various waves created at a single interface, but by following each of these waves the resultant effect of a stratum as a whole can be calculated. This latter problem has recently been given direct treatments under conditions where the media bounding the stratum are both fluid[6] or both solid elastic media.[7] And although a number of numerical calculations have been made when the elastic media bounding the stratum are the same, a complete program covering all the possible cases of this type would be so extended that it would hardly be feasible for one person or organization to carry it through. It has, therefore, seemed most reasonable to return to the problem involving only a single interface, since in principle the solution to this problem also makes possible at least step by step solutions of the more general system where a complete reflecting layer and even multiple reflecting beds have to be treated. Even so, it is recognized

[3] H. Jeffreys, M.N.R.A.S. Geoph. Suppl. *1*, 321, (1926).

[4] B. Gutenberg, Hand. d. Geophysik. IV, p. 48, 55, (1929).

[5] H. P. Berlage, Gerl. Beitr. z. Geophysik, *26*, 131, (1930).

[6] H. Reissner, Helvetica Physica Acta, *11*, 140, (1938); F. Levi and N. S. N. Nath, loc. cit. 408, (1938).

[7] M. Muskat, Geophysics, *3*, 198, (1938). Mention should also be made of the extended series of studies by K. Sezawa and K. Kanai and their collaborators, published in the Bulletins of the Earthquake Research Institute, beginning with Vol. 8, 1930, in which the analytical expressions for the reflection coefficients are derived for continuous plane waves and finite wave trains impinging upon single interfaces or reflecting strata with various types of interfacial boundary conditions.

that because of the limitation of most of the results to angles of incidence not exceeding 30° the present work is decidedly incomplete, and still leaves much to be done. However, within this range, which is also that of greatest interest in the geophysical applications pertaining to "reflection shooting," the results presented here should cover most of the situations occurring in practice.

REFLECTION AND TRANSMISSION COEFFICIENT FORMULAS

Since the general theory of the reflection of plane waves from surfaces of discontinuity has already been repeated many times in the literature, we shall reproduce here only that required for an understanding of the notation and terminology that we shall have to use. For the cases of incident longitudinal waves or transverse waves polarized in the plane of incidence we shall represent the wave systems in terms of the dilatation function Φ and shear function χ. These will satisfy the wave equations:

$$(\nabla^2 + h^2)\Phi = 0; \qquad (\nabla^2 + k^2)\chi = 0, \tag{1}*$$

$$h^2 = \omega^2/\alpha^2; \qquad k^2 = \omega^2/\beta^2, \tag{2}$$

where ω is the wave frequency, and α and β are the longitudinal and transverse wave velocities respectively.

The displacements, u, v, w, along the x, y, z, axes are given in terms of Φ and χ by:

$$u = e^{i\omega t}\left(\frac{\partial\Phi}{\partial x} + \frac{\partial^2\chi}{\partial x\partial z}\right); \qquad v = e^{i\omega t}\left(\frac{\partial\Phi}{\partial y} + \frac{\partial^2\chi}{\partial y\partial z}\right);$$
$$w = e^{i\omega t}\left(\frac{\partial\Phi}{\partial z} + \frac{\partial^2\chi}{\partial z^2} + k^2\chi\right), \tag{3}$$

t being the time variable. As the xz plane will be taken as the plane of incidence, Φ and χ will be independent of y, and v will vanish everywhere. The actual wave dilatation will be given by: $\nabla^2\Phi$ or $-h^2\Phi$, and we shall deal directly with the amplitudes of Φ and χ rather than with the displacements.

For an incident longitudinal wave, we, therefore, have the system represented by (cf. Fig. 1):

* It is assumed throughout this work that the elastic media are ideal, i.e. nondispersive and nonabsorptive.

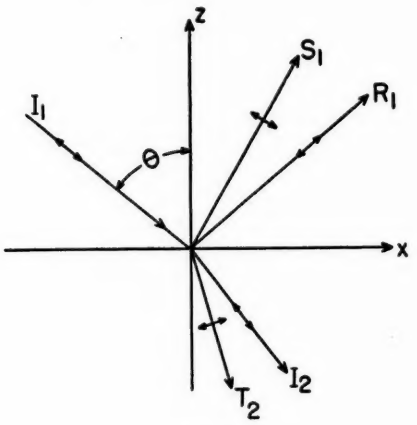

FIG. 1

$$\left.\begin{aligned} \Phi_1 &= I_1 e^{i(a_1 z - bx)} + R_1 e^{i(-a_1 z - bx)}; \\ \chi_1 &= iS_1 e^{i(-c_1 z - bx)}; \\ \Phi_2 &= I_2 e^{i(a_2 z - bx)}; \qquad \chi_2 = iT_2 e^{i(c_2 z - bx)}, \end{aligned}\right\} \tag{4}$$

where the subscripts 1 and 2 denote the incident and refracting media, respectively. The exponential constants are defined by:

$$a^2 + b^2 = h^2; \quad c^2 + b^2 = k^2; \quad b_1 = \frac{\omega \sin\theta}{\alpha_1} = b_2 \equiv b. \tag{5}$$

Applying the usual boundary conditions, we find for the "reflection" and "transmission coefficients" with respect to the amplitudes of the various waves the expressions:

$$\left.\begin{aligned} \frac{R_1}{I_1} &= \frac{\bar{f} - \bar{\tau} - \bar{\kappa}}{f + \tau + \kappa}; \qquad \frac{S_1}{I_1} = \frac{-2a_1(\nu\eta + 2a_2c_2\zeta\xi)}{f + \tau + \kappa}; \\ \frac{I_2}{I_1} &= \frac{2a_1k_1^2(c_1\nu + c_2\xi)}{f + \tau + \kappa}; \qquad \frac{T_2}{I_1} = \frac{2a_1k_1^2(2a_2c_1\zeta - \eta)}{f + \tau + \kappa}, \end{aligned}\right\} \tag{6}$$

where:

$$\left.\begin{aligned} \bar{f} &= k_1^2k_2^2\delta(a_1c_2 \mp a_2c_1); \quad \bar{\tau} = b^2\eta^2 \mp a_1c_1\nu^2; \\ \bar{\kappa} &= a_2c_2(\xi^2 \mp 4a_1c_1\zeta^2b^2); \quad \delta = \mu_2/\mu_1; \quad \zeta = \delta - 1; \\ \eta &= \delta\epsilon_2 - \epsilon_1; \quad \nu = \delta\epsilon_2 + 2b^2; \quad \xi = \epsilon_1 + 2\delta b^2; \quad \epsilon = k^2 - 2b^2; \end{aligned}\right\} \tag{7}$$

and μ is the modulus of rigidity.

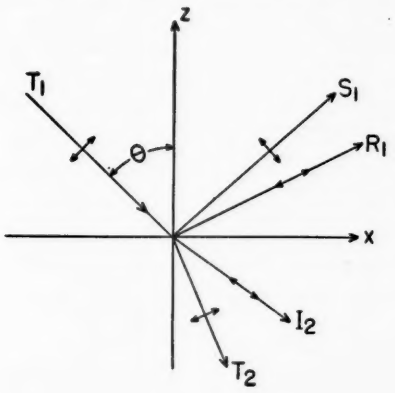

FIG. 2

With the same notation and beginning with an incident transverse wave (cf. Fig. 2), the wave system will be represented by the functions:

$$\left.\begin{aligned} \Phi_1 &= R_1 e^{i(-a_1 z - bx)}; \\ \chi_1 &= iT_1 e^{i(c_1 z - bx)} + iS_1 e^{i(-c_1 z - bx)}; \\ \Phi_2 &= I_2 e^{i(a_2 z - bx)}; \qquad \chi_2 = iT_2 e^{i(c_2 z - bx)}. \end{aligned}\right\} \tag{8}$$

The reflection and transmission coefficients for this system may be shown to be:

$$\left.\begin{aligned} \frac{R_1}{T_1} &= \frac{2c_1 b^2(\nu\eta + 2a_2 c_2 \zeta\xi)}{f + \tau + \kappa}; & \frac{S_1}{T_1} &= -\frac{(\bar{f} + \bar{\tau} + \bar{\kappa})}{f + \tau + \kappa}; \\ \frac{I_2}{T_1} &= -\frac{2c_1 k_1^2 b^2(2a_1 c_2 \zeta - \eta)}{f + \tau + \kappa}; & \frac{T_2}{T_1} &= \frac{2c_1 k_1^2(a_2\xi + a_1\nu)}{f + \tau + \kappa}. \end{aligned}\right\} \tag{9}$$

To obtain the corresponding coefficients in terms of the amplitudes of the displacements in the various wave components it is only necessary to note that the resultant displacement amplitude of a longitudinal wave will be I or R multiplied by h, while that of a transverse wave will be S or T multiplied by bk.

The energy fluxes per unit section normal to the direction of propagation will be $\gamma\omega^4/2\alpha$ multiplied into I^2 or R^2 for a longitudinal wave, and $\gamma\omega^4 b^2/2\beta$ multiplied into S^2 or T^2 for a transverse wave, γ being the density of the medium. Hence the coefficients expressed in terms of unit incident energy flux will be for an incident longitudinal wave:

$$\left.\begin{aligned} L_r &= \frac{R_1^2}{I_1^2} = \text{longitudinal wave reflection coefficient,} \\ T_r &= \frac{\alpha_1 b^2}{\beta_1}\frac{S_1^2}{I_1^2} = \text{transverse wave reflection coefficient,} \\ L_t &= \frac{\gamma_2\alpha_1}{\gamma_1\alpha_2}\frac{I_2^2}{I_1^2} = \text{longitudinal wave transmission coefficient,} \\ T_t &= \frac{\gamma_2\alpha_1 b^2}{\gamma_1\beta_2}\frac{T_2^2}{I_1^2} = \text{transverse wave transmission coefficient.} \end{aligned}\right\} \quad (10)$$

For an incident transverse wave we shall have similarly:

$$\left.\begin{aligned} L_r &= \frac{\beta_1}{\alpha_1 b^2}\frac{R_1^2}{T_1^2} = \text{longitudinal wave reflection coefficient,} \\ T_r &= \frac{S_1^2}{T_1^2} = \text{transverse wave reflection coefficient,} \\ L_t &= \frac{\gamma_2\beta_1}{\gamma_1\alpha_2 b^2}\frac{I_2^2}{T_1^2} = \text{longitudinal wave transmission coefficient,} \\ T_t &= \frac{\gamma_2\beta_1}{\gamma_1\beta_2}\frac{T_2^2}{T_1^2} = \text{transverse wave transmission coefficient.} \end{aligned}\right\} \quad (11)$$

While the energy density fluxes given by Eqs. (10) and (11) completely describe the energy distribution in the elastic media from a strictly physical point of view, there is also considerable practical interest in the fractions of the incident energy carried away by the various waves. The values of these are to be obtained from the intensities of the flux, given by Eqs. (10) and (11), by correcting them for the different widths of the various wave beams resulting from a unit width of the incident beam. For a wave leaving the interface at an angle $\bar{\theta}$ with the normal the factor is easily shown to be $\cos\bar{\theta}/\cos\theta$, θ being the angle of incidence. Denoting by the barred letters these reflection coefficients, we thus have, finally:

Incident longitudinal waves:

$$\left.\begin{aligned}
\overline{L}_r &= \frac{R_1^2}{I_1^2} = \text{longitudinal wave reflection coefficient} = L_r,\\
\overline{T}_r &= \frac{c_1 b^2}{a_1}\,\frac{S_1^2}{I_1^2} = \text{transverse wave reflection coefficient},\\
\overline{L}_t &= \frac{\gamma_2 a_2}{\gamma_1 a_1}\,\frac{I_2^2}{I_1^2} = \text{longitudinal wave transmission coefficient},\\
\overline{T}_2 &= \frac{\gamma_2 c_2 b^2}{\gamma_1 a_1}\,\frac{T_2^2}{I_1^2} = \text{transverse wave transmission coefficient}.
\end{aligned}\right\} \quad (12)$$

Incident transverse waves:

$$\left.\begin{aligned}
\overline{L}_r &= \frac{a_1}{c_1 b^2}\,\frac{R_1^2}{T_1^2} = \text{longitudinal wave reflection coefficient},\\
\overline{T}_r &= \frac{S_1^2}{T_1^2} = \text{transverse wave reflection coefficient} = T_r,\\
\overline{L}_t &= \frac{\gamma_2 a_2}{\gamma_1 c_1 b^2}\,\frac{I_2^2}{T_1^2} = \text{longitudinal wave transmission coefficient},\\
\overline{T}_t &= \frac{\gamma_2 c_2}{\gamma_1 c_1}\,\frac{T_2^2}{T_1^2} = \text{transverse wave transmission coefficient}.
\end{aligned}\right\} \quad (13)$$

It should be noted that both the coefficients of Eqs. (10) and (11) and those of Eqs. (12) and (13) apply also to representations in which the incident wave displacements are specified rather than the amplitude functions Φ or χ. Moreover, by virtue of their definition, the sum of the coefficients of Eqs. (12) or (13) must equal unity, i.e.:

$$\overline{L}_r + \overline{T}_r + \overline{L}_t + \overline{T}_t = 1. \quad (14)$$

This relation serves as a useful check on the calculations. For the results reported below the accuracy was such that in no case did the left side of Eq. (14) differ from unity by more than 0.0001. One may, therefore, have faith in the correctness to this accuracy of the individual values, and for this reason they will be recorded in detail in tabular form.

For the simple case where the incident transverse waves are polarized normal to the plane of incidence the amplitude coefficients are given directly by:

$$v_r/v_1 = (c_1 - c_2\delta)/(c_1 + c_2\delta); \qquad v_2/v_1 = 2c_1/(c_1 + c_2\delta), \quad (15)$$

where v_r, v_1, v_2 are the displacement amplitudes of the reflected, incident, and transmitted waves, and c_1, c_2, δ are defined by Eqs. (5) and (7). The wave intensities are then $\gamma\beta\omega^2v^2/2$, γ being the density of the medium, and the reflection coefficients representing the fraction of the incident energy which is reflected are simply v_r^2/v_1^2. These may be expressed directly in terms of the angle of incidence θ, the "velocity contrast" $\bar{\beta}=\beta_2/\beta_1$, and "density contrast" $\bar{\gamma}=\gamma_2/\gamma_1$ by:

$$\frac{v_r^2}{v_1^2}=\left(\frac{1-\bar{\beta}\bar{\gamma}\sqrt{\bar{\beta}^2+(1-\bar{\beta}^2)\sec^2\theta}}{1+\bar{\beta}\bar{\gamma}\sqrt{\bar{\beta}^2+(1-\bar{\beta}^2)\sec^2\theta}}\right)^2. \tag{16}$$

As there are no longitudinal waves or transverse waves polarized in the plane of incidence created at the interfaces, the transmission coefficient, giving the fraction of the incident energy which is transmitted, will be simply $1-v_r^2/v_1^2$.

NUMERICAL RESULTS

All the numerical calculations for the cases of incident longitudinal waves or incident transverse waves polarized in the plane of incidence have been carried through under the assumption that in both the incident and refracting medium the Poisson ratio has the value $1/4$, so that $\alpha/\beta=\sqrt{3}$. For the case of incident longitudinal waves the calculations have been carried through for angles of incidence, θ, from 0 to 30° in steps of 5°. The "velocity contrast" $\bar{\alpha}\equiv\alpha_2/\alpha_1$ has been varied from 0.5 to 2.0 in steps of 0.25, and the "density contrast" $\bar{\gamma}\equiv\gamma_2/\gamma_1$ from 0.7 to 1.3 in steps of 0.1. These ranges exclude the region of total reflection except for the single angle $\theta=30°$ and velocity contrast $\bar{\alpha}=2.0$, which are critical conditions for the longitudinal waves. However, as is well known, the formal expressions for the reflection coefficients, such as given by Eqs. (6) and (9), are also valid in the region of total reflection, if proper account is taken of the complex character of the various amplitudes.

In the case of incident transverse waves polarized in the plane of incidence, no direct calculations have as yet been made for evenly spaced angles of incidence. Rather the computations were made for those angles of incidence which gave the same preliminary constants of Eq. (7) as those applying to the various cases of a longitudinal incident wave, i.e. at angles $\theta=\sin^{-1}\beta/\alpha\,\sin\theta_1$, where θ_1 were the angles of incidence used in the calculations for incident longitudinal waves. As these angles were spaced by approximately $2\frac{1}{2}°$ for the 5° intervals

of the longitudinal incident wave calculations, only every other set was carried through. While the odd values of the angles detracts from the value of the calculations for a systematic tabulation, they are, of course, just as satisfactory for graphical uses as would be any evenly spaced set of angles of incidence.

For incident transverse waves polarized normal to the plane of incidence, the calculations have been carried through completely for all angles for the velocity contrasts betweeen 0.50 and 2.0 and density contrasts between 0.7 and 1.3. While this range of angles extends to 90° for $\bar{\beta} \leqq 1$, they were formally dropped, for $\bar{\beta} > 1$, at the critical angles given by:

$$\theta_c = \sin^{-1} 1/\bar{\beta},$$

since for all angles $\theta > \theta_c$ the reflection coefficients have the value unity for all $\bar{\gamma}$.

The numerical values of the various reflection coefficients are given in Tables I, II, and III for incident longitudinal waves, in Tables IV, V, and VI for incident transverse waves polarized in the plane of incidence, and in Table VII for incident transverse waves polarized normal to the plane of incidence. They are listed to four decimal places, although the preliminary constants were calculated to five decimal places and sometimes even to six. Each table consists of seven separate tabulations for each of the values of the density contrast, $\bar{\gamma}$.

In order to give a qualitative graphical idea of the variations in the reflection and transmission coefficients with the physical interfacial parameters, a condensed plot of the energy fractions for incident longitudinal waves, as given in Table III, is given in Fig. 3. Only the data for $\bar{\gamma} = 0.7$ and 1.3 have been plotted, so as to avoid excessive overlapping. The reader may plot for himself the data of the other tables. If interpolation is desired other types of plotting may be used.

It will be clear from the multiplicity of intermediate numerical steps required in the calculation of any specific reflection coefficient that no simple physical interpretation or explanation can be given for the manifold variations of the coefficients with the various parameters. The final results must be accepted simply as numerical implications of elastic wave theory. Even on plotting characteristic features stand out only in the grossest manner. Thus, for example, if the coefficients are plotted vs. $\bar{\alpha}$, one finds a rough parallelism of the various reflection coefficient curves for different values of $\bar{\gamma}$, a sym-

metry of position with respect to the case of $\bar{\gamma}=1.0$, an approximate symmetry of the intensity curves with respect to the value $\bar{\alpha}=1.0$, and finally steep variations in the values of the coefficients as the critical conditions for total reflection are approached, namely: $\theta=30°$, $\bar{\alpha}=2.0$ for incident longitudinal waves, $\theta=16°46'45''$, $\bar{\alpha}=2.0$ for transverse waves polarized in the plane of incidence, or $\theta=\sin^{-1}1/\bar{\beta}$ for incident transverse waves polarized normal to the plane of incidence. Fig. 3 shows even more directly the high sensitivity of the coefficients to the physical parameters in the vicinity of total reflection conditions.

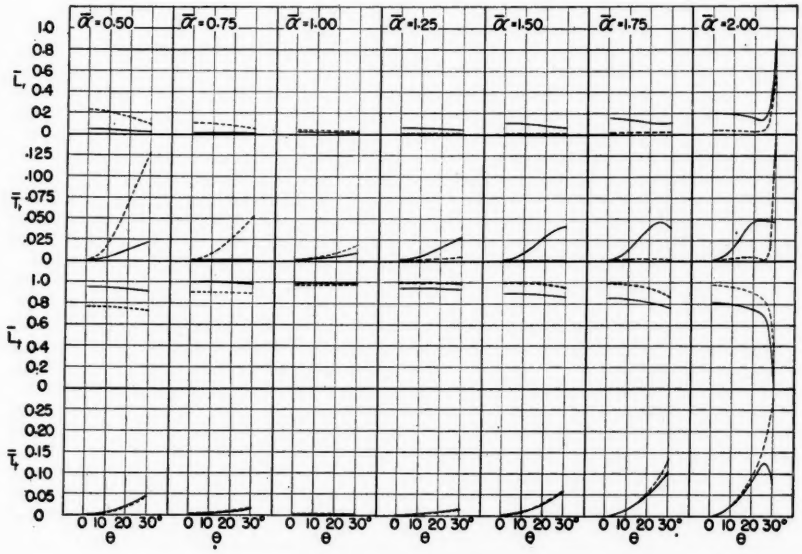

FIG. 3. The fractions of the incident longitudinal wave energy transformed into reflected and transmitted waves as functions of the angle of incidence, θ.

$\bar{L}_r, \bar{T}_r, \bar{T}_t, \bar{L}_t$ = fractions of incident energy converted into reflected longitudinal, reflected transverse, transmitted longitudinal, and transmitted transverse waves, respectively.

$\bar{\alpha}$ = (velocity in refracting medium)/(velocity in incident medium).

(density of refracting medium)/(density of incident medium) = 0.7 for dotted curves = 1.3 for solid curves.

Some applications of these tables to certain typical reflection shooting problems will be given in a following paper.

The authors* wish to thank Dr. Paul D. Foote and Dr. E. A. Eckhardt of this laboratory for permission to publish this paper.

* I regret to announce the death of M. W. Meres at Pittsburgh, Feb. 20, 1940.—EDITOR.

TABLE 1

AMPLITUDES OF THE LONGITUDINAL AND TRANSVERSE WAVES CREATED AT AN ELASTIC INTERFACE AFTER INCIDENCE OF A LONGITUDINAL WAVE

$\bar{\gamma}=\gamma_2/\gamma_1=0.7$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$		0.75	1.00	1.25	1.50	1.75	2.00
0°	R_1/I_1	−0.4815	−0.3115	−0.1765	−0.0667	0.0244	0.1012	0.1667
	S_1/I_1	0.6368	0.4062	0.2223	0.0722	−0.0526	−0.1580	−0.2483
	I_2/I_1	0.7407	0.9836	1.1765	1.3333	1.4634	1.5731	1.6667
	T_2/I_1	−0.2543	−0.1769	−0.0185	0.1984	0.4591	0.7534	1.0739
5°	R_1/I_1	−0.4758	−0.3079	−0.1746	−0.0663	0.0235	0.0991	0.1636
	S_1/I_1	0.6333	0.4040	0.2214	0.0728	−0.0506	−0.1544	−0.2431
	I_2/I_1	0.7392	0.9826	1.1765	1.3346	1.4660	1.5770	1.6720
	T_2/I_1	−0.2543	−0.1769	−0.0185	0.1988	0.4603	0.7557	1.0780
10°	R_1/I_1	−0.4591	−0.2974	−0.1692	−0.0651	0.0210	0.0934	0.1552
	S_1/I_1	0.6227	0.3973	0.2188	0.0744	−0.0443	−0.1434	−0.2268
	I_2/I_1	0.7345	0.9796	1.1764	1.3384	1.4741	1.5898	1.6898
	T_2/I_1	−0.2542	−0.1770	−0.0182	0.2002	0.4639	0.7630	1.0906
15°	R_1/I_1	−0.4323	−0.2805	−0.1603	−0.0629	0.0178	0.0858	0.1442
	S_1/I_1	0.6053	0.3863	0.2145	0.0773	−0.0317	−0.1238	−0.1968
	I_2/I_1	0.7266	0.9745	1.1764	1.3452	1.4892	1.6150	1.7272
	T_2/I_1	−0.2541	−0.1772	−0.0179	0.2025	0.4702	0.7759	1.1134
20°	R_1/I_1	−0.3967	−0.2581	−0.1483	−0.0589	0.0159	0.0802	0.1383
	S_1/I_1	0.5816	0.3712	0.2086	0.0814	−0.0180	−0.0935	−0.1464
	I_2/I_1	0.7156	0.9672	1.1764	1.3557	1.5144	1.6606	1.8045
	T_2/I_1	−0.2539	−0.1774	−0.0173	0.2059	0.4796	0.7960	1.1497
25°	R_1/I_1	−0.3544	−0.2316	−0.1335	−0.0522	0.0187	0.0862	0.1639
	S_1/I_1	0.5520	0.3523	0.2011	0.0868	0.0041	−0.0465	−0.0512
	I_2/I_1	0.7015	0.9575	1.1764	1.3714	1.5555	1.7478	1.9928
	T_2/I_1	−0.2534	−0.1774	−0.0166	0.2105	0.4933	0.8266	1.2102
30°	R_1/I_1	−0.3079	−0.2025	−0.1164	−0.0411	0.0338	0.1350	0.7595
	S_1/I_1	0.5173	0.3301	0.1921	0.0939	0.0352	0.0370	0.5554
	I_2/I_1	0.6843	0.9452	1.1764	1.3947	1.6264	1.9472	3.6623
	T_2/I_1	−0.2526	−0.1773	−0.0155	0.2162	0.5133	0.8790	1.3885

$\bar{\gamma}$ = (density of refracting medium)/(density of incident medium) = γ_2/γ_1.

$\bar{\alpha}$ = (velocity in refracting medium)/(velocity in incident medium) = α_2/α_1.

R_1, S_1, I_2, T_2 = amplitudes of wave functions for reflected longitudinal and transverse waves, and transmitted longitudinal and transverse waves.

I_1 = amplitude of incident longitudinal wave function; θ = angle of incidence.

TABLE I—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=0.8$

θ		$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	R_1/I_1	−0.4286	−0.2500	−0.1111	0.0000	0.0909	0.1667	0.2308
	S_1/I_1	0.5686	0.3270	0.1393	−0.0112	−0.1345	−0.2374	−0.3246
	I_2/I_1	0.7143	0.9375	1.1111	1.2500	1.3636	1.4584	1.5385
	T_2/I_1	−0.2426	−0.1641	−0.0110	0.1944	0.4793	0.7102	1.0045
5°	R_1/I_1	−0.4235	−0.2471	−0.1100	−0.0003	0.0893	0.1640	0.2271
	S_1/I_1	0.5645	0.3251	0.1388	−0.0103	−0.1322	−0.2336	−0.3193
	I_2/I_1	0.7128	0.9366	1.1111	1.2512	1.3660	1.4620	1.5433
	T_2/I_1	−0.2426	−0.1641	−0.0110	0.1947	0.4389	0.7122	1.0080
10°	R_1/I_1	−0.4085	−0.2386	−0.1065	−0.0012	0.0849	0.1565	0.2170
	S_1/I_1	0.5547	0.3194	0.1372	−0.0077	−0.1252	−0.2220	−0.3027
	I_2/I_1	0.7082	0.9337	1.1111	1.2547	1.3736	1.4738	1.5597
	T_2/I_1	−0.2425	−0.1642	−0.0108	0.1958	0.4418	0.7182	1.0186
15°	R_1/I_1	−0.3845	−0.2249	−0.1010	−0.0022	0.0785	0.1459	0.2031
	S_1/I_1	0.5385	0.3100	0.1345	0.0033	−0.1131	−0.2014	−0.2722
	I_2/I_1	0.7007	0.9288	1.1111	1.2611	1.3877	1.4972	1.5943
	T_2/I_1	−0.2424	−0.1643	−0.0106	0.1975	0.4462	0.7288	1.0375
20°	R_1/I_1	−0.3528	−0.2068	−0.0935	−0.0027	0.0722	0.1360	0.1929
	S_1/I_1	0.5166	0.2970	0.1308	0.0032	−0.0955	−0.1696	−0.2210
	I_2/I_1	0.6901	0.9219	1.1111	1.2710	1.4112	1.5398	1.6665
	T_2/I_1	−0.2420	−0.1644	−0.0103	0.2001	0.4544	0.7451	1.0668
25°	R_1/I_1	−0.3153	−0.1857	−0.0843	−0.0016	0.0695	0.1364	−0.2126
	S_1/I_1	0.4892	0.2808	0.1262	0.0112	−0.0710	−0.1206	−0.1244
	I_2/I_1	0.6765	0.9126	1.1111	1.2858	1.4500	1.6220	1.8443
	T_2/I_1	−0.2414	−0.1645	−0.0098	0.2036	0.4651	0.7690	1.1122
30°	R_1/I_1	−0.2742	−0.1628	−0.0736	+0.0031	+0.0782	+0.1784	+0.8088
	S_1/I_1	0.4572	0.2618	0.1206	0.0217	−0.0367	−0.0338	0.5064
	I_2/I_1	0.6599	0.9009	1.1111	1.3079	1.5172	1.8120	3.4749
	T_2/I_1	−0.2404	−0.1643	−0.0092	0.2081	0.4803	0.8073	1.1686
		$\bar{\gamma}=\gamma_2/\gamma_1=0.9$						
0°	R_1/I_1	−0.3793	−0.1940	−0.0526	0.0588	0.1489	0.2233	0.2857
	S_1/I_1	0.5038	0.2555	0.0657	−0.0840	−0.2051	−0.3151	−0.3891
	I_2/I_1	0.6897	0.8955	1.0526	1.1764	1.2766	1.3592	1.4285
	T_2/I_1	−0.2320	−0.1528	−0.0050	0.1899	0.4183	0.6713	0.9432
5°	R_1/I_1	−0.3748	−0.1917	−0.0521	0.0579	0.1468	0.2202	0.2816
	S_1/I_1	0.5008	0.2538	0.0655	−0.0829	−0.2026	−0.3012	−0.3838
	I_2/I_1	0.6882	0.8946	1.0526	1.1775	1.2788	1.3626	1.4330
	T_2/I_1	−0.2320	−0.1529	−0.0049	0.1903	0.4190	0.6731	0.9462
10°	R_1/I_1	−0.3614	−0.1850	−0.0505	0.0553	0.1407	0.2111	0.2700
	S_1/I_1	0.4917	0.2490	0.0647	−0.0795	−0.1949	−0.2891	−0.3670
	I_2/I_1	0.6838	0.8919	1.0526	1.1809	1.2859	1.3736	1.4482
	T_2/I_1	−0.2318	−0.1529	−0.0049	0.1909	0.4213	0.6782	0.9555
15°	R_1/I_1	−0.3400	−0.1745	−0.0479	0.0514	0.1316	0.1978	0.2537
	S_1/I_1	0.4768	0.2408	0.0635	−0.0737	−0.1802	−0.2679	−0.3363
	I_2/I_1	0.6766	0.8872	1.0526	1.1869	1.2991	1.3953	1.4804
	T_2/I_1	−0.2316	−0.1530	−0.0048	0.1923	0.4256	0.6870	0.9713
20°	R_1/I_1	−0.3118	−0.1600	−0.0444	0.0470	0.1194	0.1843	0.2398
	S_1/I_1	0.4564	0.2299	0.0581	−0.0657	−0.1620	−0.2351	−0.2848
	I_2/I_1	0.6664	0.8806	1.0527	1.1962	1.3214	1.4354	1.5480
	T_2/I_1	−0.2312	−0.1531	−0.0046	0.1942	0.4343	0.7002	0.9952
25°	R_1/I_1	−0.2787	−0.1437	−0.0400	0.0433	0.1139	0.1799	0.2542
	S_1/I_1	0.4312	0.2161	0.0596	−0.0551	−0.1362	−0.1847	−0.1877
	I_2/I_1	0.6533	0.8718	1.0527	1.2103	1.3578	1.5131	1.7165
	T_2/I_1	−0.2304	−0.1531	−0.0044	0.1967	0.4398	0.7188	1.0286
30°	R_1/I_1	−0.2427	−0.1263	−0.0350	0.0423	0.1170	0.2157	0.8479
	S_1/I_1	0.4015	0.1999	0.0570	−0.0418	−0.0994	−0.0956	0.4610
	I_2/I_1	0.6372	0.8605	1.0526	1.2313	1.4219	1.6946	3.3105
	T_2/I_1	−0.2293	−0.1529	−0.0041	0.1999	0.4513	0.7462	0.9880

TABLE 1—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.0$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	R_1/I_1	−0.3333	−0.1429	0	0.1111	0.2000	0.2728	0.3333
	S_1/I_1	0.4444	0.1905	0	−0.1482	−0.2667	−0.3636	−0.4444
	I_2/I_1	0.6667	0.8571	1	1.1111	1.2000	1.2728	1.3333
	T_2/I_1	−0.2222	−0.1429	0	0.1852	0.4000	0.6363	0.8889
5°	R_1/I_1	−0.3293	−0.1411	0	0.1097	0.1974	0.2692	0.3288
	S_1/I_1	0.4416	0.1891	0	−0.1468	−0.2640	−0.3595	−0.4390
	I_2/I_1	0.6653	0.8563	1	1.1121	1.2020	1.2759	1.3375
	T_2/I_1	−0.2222	−0.1429	0	0.1854	0.4006	0.6378	0.8915
10°	R_1/I_1	−0.3174	−0.1359	0	0.1056	0.1898	0.2587	0.3160
	S_1/I_1	0.4332	0.1850	0	−0.1427	−0.2558	−0.3472	−0.4222
	I_2/I_1	0.6611	0.8537	1	1.1153	1.2087	1.2861	1.3515
	T_2/I_1	−0.2220	−0.1429	0	0.1860	0.4027	0.6422	0.8994
15°	R_1/I_1	−0.2984	−0.1277	0	0.0991	0.1784	0.2433	0.2976
	S_1/I_1	0.4194	0.1782	0	−0.1359	−0.2419	−0.3255	−0.3914
	I_2/I_1	0.6540	0.8493	1	1.1209	1.2211	1.3065	1.3817
	T_2/I_1	−0.2217	−0.1430	0	0.1870	0.4060	0.6496	0.9131
20°	R_1/I_1	−0.2735	−0.1172	0	0.0913	0.1652	0.2266	0.2807
	S_1/I_1	0.4006	0.1689	0	−0.1263	−0.2215	−0.2919	−0.3400
	I_2/I_1	0.6442	0.8429	1	1.1298	1.2419	1.3442	1.4453
	T_2/I_1	−0.2212	−0.1430	0	0.1884	0.4107	0.6604	0.9325
25°	R_1/I_1	−0.2444	−0.1052	0	0.0833	0.1533	0.2179	0.2903
	S_1/I_1	0.3772	0.1573	0	−0.1138	−0.1935	−0.2406	−0.2430
	I_2/I_1	0.6315	0.8345	1	1.1431	1.2767	1.4179	1.6053
	T_2/I_1	−0.2204	−0.1430	0	0.1901	0.4172	0.6749	0.9567
30°	R_1/I_1	−0.2131	−0.0928	0	0.0774	0.1513	0.2481	0.8791
	S_1/I_1	0.3499	0.1437	0	−0.0981	−0.1547	−0.1502	0.4188
	I_2/I_1	0.6160	0.8236	1	1.1632	1.3377	1.5915	3.1659
	T_2/I_1	−0.2191	0.1428	0	0.1923	0.4256	0.6935	0.8376
				$\bar{\gamma}=\gamma_2/\gamma_1=1.1$				
0°	R_1/I_1	−0.2903	−0.0959	0.0476	0.1579	0.2453	0.3163	0.3750
	S_1/I_1	0.3891	0.1312	−0.0590	−0.2051	−0.3208	−0.4146	−0.4923
	I_2/I_1	0.6452	0.8219	0.9524	1.0526	1.1321	1.1966	1.2500
	T_2/I_1	−0.2132	−0.1340	0.0040	0.1804	0.3828	0.6047	0.8403
5°	R_1/I_1	−0.2867	−0.0946	0.0471	0.1560	0.2422	0.3123	0.3701
	S_1/I_1	0.3865	0.1300	−0.0588	−0.2035	−0.3179	−0.4104	−0.4869
	I_2/I_1	0.6438	0.8211	0.9524	1.0536	1.1339	1.1995	1.2538
	T_2/I_1	−0.2131	−0.1340	0.0040	0.1805	0.3833	0.6059	0.8426
10°	R_1/I_1	−0.2762	−0.0909	0.0457	0.1506	0.2335	0.3007	0.3564
	S_1/I_1	0.3774	0.1266	−0.0582	−0.1989	−0.3093	−0.3979	−0.4701
	I_2/I_1	0.6398	0.8186	0.9524	1.0566	1.1402	1.2091	1.2670
	T_2/I_1	−0.2130	−0.1340	0.0040	0.1810	0.3850	0.6096	0.8495
15°	R_1/I_1	−0.2595	−0.0851	0.0434	0.1419	0.2220	0.2834	0.3362
	S_1/I_1	0.3660	0.1210	−0.0570	−0.1912	−0.2947	−0.3758	−0.4394
	I_2/I_1	0.6330	0.8144	0.9524	1.0619	1.1519	1.2283	1.2953
	T_2/I_1	−0.2126	−0.1341	0.0039	0.1817	0.3877	0.6160	0.8612
20°	R_1/I_1	−0.2382	−0.0777	0.0402	0.1311	0.2038	0.2639	0.3165
	S_1/I_1	0.3490	0.1133	−0.0555	−0.1802	−0.2734	−0.3418	−0.3882
	I_2/I_1	0.6234	0.8083	0.9524	1.0703	1.1716	1.2639	1.3553
	T_2/I_1	−0.2119	−0.1341	0.0038	0.1826	0.3915	0.6249	0.8772
25°	R_1/I_1	−0.2123	−0.0697	0.0364	0.1193	0.1883	0.2515	0.3218
	S_1/I_1	0.3270	0.1037	−0.0536	−0.1661	−0.2441	−0.2899	−0.2917
	I_2/I_1	0.6112	0.8002	0.9524	1.0830	1.2047	1.3339	1.5078
	T_2/I_1	−0.2111	−0.1340	0.0036	0.1838	0.3964	0.6360	0.8941
30°	R_1/I_1	−0.1853	−0.0619	0.0318	0.1090	0.1818	0.2765	0.9042
	S_1/I_1	0.3019	0.0924	−0.0513	−0.1484	−0.2038	−0.1986	0.3793
	I_2/I_1	0.5962	0.7898	0.9524	1.1021	1.2630	1.5003	3.0360
	T_2/I_1	−0.2098	−0.1338	0.0034	0.1851	0.4024	0.6476	0.7112

TABLE I—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.2$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	R_1/I_1	−0.2500	−0.0526	0.0909	0.2000	0.2857	0.3549	0.4118
	S_1/I_1	0.3375	0.0769	−0.1124	−0.2560	−0.3686	−0.4594	−0.5342
	I_2/I_1	0.6250	0.7895	0.9091	1.0000	1.0714	1.1291	1.1765
	T_2/I_1	−0.2048	−0.1260	0.0074	0.1756	0.3674	0.5761	0.7967
5°	R_1/I_1	−0.2468	−0.0518	0.0900	0.1977	0.2823	0.3506	0.4067
	S_1/I_1	0.3351	0.0759	−0.1119	−0.2542	−0.3657	−0.4552	−0.5288
	I_2/I_1	0.6237	0.7887	0.9090	1.0009	1.0732	1.1318	1.1800
	T_2/I_1	−0.2048	−0.1260	0.0074	0.1757	0.3678	0.5772	0.7987
10°	R_1/I_1	−0.2376	−0.0494	0.0873	0.1911	0.2724	0.3380	0.3919
	S_1/I_1	0.3279	0.0731	−0.1107	−0.2491	−0.3568	−0.4425	−0.5120
	I_2/I_1	0.6198	0.7863	0.9091	1.0038	1.0791	1.1408	1.1924
	T_2/I_1	−0.2046	−0.1261	0.0073	0.1760	0.3692	0.5804	0.8047
15°	R_1/I_1	−0.2229	−0.0458	0.0829	0.1805	0.2572	0.3190	0.3702
	S_1/I_1	0.3161	0.0685	−0.1086	−0.2406	−0.3416	−0.4202	−0.4815
	I_2/I_1	0.6132	0.7822	0.9091	1.0088	1.0901	1.1589	1.2191
	T_2/I_1	−0.2042	−0.1261	0.0071	0.1765	0.3714	0.5858	0.8150
20°	R_1/I_1	−0.2039	−0.0413	0.0769	0.1670	0.2384	0.2972	0.3481
	S_1/I_1	0.3001	0.0623	−0.1057	−0.2285	−0.3195	−0.3858	−0.4306
	I_2/I_1	0.6040	0.7764	0.9091	1.0168	1.1089	1.1926	1.2758
	T_2/I_1	−0.2035	−0.1261	0.0069	0.1771	0.3744	0.5931	0.8280
25°	R_1/I_1	−0.1820	−0.0369	0.0695	0.1519	0.2197	0.2814	0.3496
	S_1/I_1	0.2802	0.0545	−0.1021	−0.2129	−0.2892	−0.3336	−0.3349
	I_2/I_1	0.5921	0.7686	0.9091	1.0289	1.1403	1.2593	1.4214
	T_2/I_1	−0.2026	−0.1260	0.0066	0.1778	0.3781	0.6015	0.8392
30°	R_1/I_1	−0.1591	−0.0333	0.0609	0.1375	0.2091	0.3015	0.9245
	S_1/I_1	0.2570	0.0454	−0.0978	−0.1935	−0.2476	−0.2420	0.3422
	I_2/I_1	0.5775	0.7586	0.9091	1.0472	1.1962	1.4191	2.9189
	T_2/I_1	−0.2012	−0.1258	0.0062	0.1784	0.3820	0.6075	0.6038

$\bar{\gamma}=\gamma_2/\gamma_1=1.3$

0°	R_1/I_1	−0.2121	−0.0127	0.1304	0.2381	0.3220	0.3893	0.4444
	S_1/I_1	0.2892	0.0269	−0.1608	−0.3017	−0.4114	−0.4992	−0.5711
	I_2/I_1	0.6061	0.7595	0.8696	0.9524	1.0170	1.0688	1.1111
	T_2/I_1	−0.1971	−0.1188	0.0101	0.1709	0.3528	0.5496	0.7573
5°	R_1/I_1	−0.2093	−0.0122	0.1291	0.2355	0.3183	0.3847	0.4390
	S_1/I_1	0.2870	0.0262	−0.1602	−0.2998	−0.4083	−0.4949	−0.5657
	I_2/I_1	0.6048	0.7587	0.8696	0.9532	1.0187	1.0713	1.1145
	T_2/I_1	−0.1970	−0.1189	0.0101	0.1710	0.3532	0.5506	0.7591
10°	R_1/I_1	−0.2013	−0.0111	0.1253	0.2278	0.3075	0.3714	0.42ε6
	S_1/I_1	0.2804	0.0240	−0.1584	−0.2943	−0.3991	−0.4820	−0.5490
	I_2/I_1	0.6010	0.7564	0.8696	0.9559	1.0242	1.0798	1.1261
	T_2/I_1	−0.1968	−0.1189	0.0100	0.1711	0.3543	0.5533	0.7645
15°	R_1/I_1	−0.1886	−0.0094	0.1190	0.2154	0.2906	0.3509	0.4005
	S_1/I_1	0.2695	0.0203	−0.1554	−0.2850	−0.3835	−0.4596	−0.5187
	I_2/I_1	0.5946	0.7526	0.8696	0.9607	1.0347	1.0969	1.1513
	T_2/I_1	−0.1964	−0.1189	0.0098	0.1714	0.3561	0.5580	0.7734
20°	R_1/I_1	−0.1722	−0.0076	0.1104	0.1995	0.2696	0.3269	0.3763
	S_1/I_1	0.2547	0.0154	−0.1513	−0.2720	−0.3607	−0.4250	−0.4682
	I_2/I_1	0.5857	0.7469	0.8696	0.9683	1.0524	1.1289	1.2052
	T_2/I_1	−0.1957	−0.1189	0.0095	0.1718	0.3584	0.5641	0.7842
25°	R_1/I_1	−0.1536	−0.0065	0.0999	0.1814	0.2481	0.3081	0.3742
	S_1/I_1	0.2364	0.0093	−0.1462	−0.2551	−0.3296	−0.3727	−0.3736
	I_2/I_1	0.5742	0.7394	0.8696	0.9799	1.0825	1.1926	1.3444
	T_2/I_1	−0.1947	−0.1188	0.0090	0.1720	0.3611	0.5702	0.7905
30°	R_1/I_1	−0.1344	−0.0067	0.0878	0.1635	0.2337	0.3239	0.9409
	S_1/I_1	0.2151	0.0022	−0.1400	−0.2342	−0.2870	−0.2810	0.3072
	I_2/I_1	0.5600	0.7298	0.8696	0.9975	1.1360	1.3462	2.8124
	T_2/I_1	−0.1932	−0.1185	0.0085	0.1720	0.3635	0.5717	0.5120

TABLE 2

INTENSITIES OF THE LONGITUDINAL AND TRANSVERSE WAVES CREATED AT AN ELASTIC INTERFACE AFTER INCIDENCE OF A LONGITUDINAL WAVE

$\bar{\gamma}=\gamma_2/\gamma_1=0.7$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$		0.75	1.00	1.25	1.50	1.75	2.00
0°	L_r	0.2318	0.0970	0.0311	0.0044	0.0006	0.0102	0.0278
	T_r	0	0	0	0	0	0	0
	L_t	0.7682	0.9030	0.9689	0.9956	0.9994	0.9899	0.9722
	T_t	0	0	0	0	0	0	0
5°	L_r	0.2264	0.0948	0.0305	0.0044	0.0006	0.0098	0.0268
	T_r	0.0053	0.0021	0.0006	0.0001	0.0000	0.0003	0.0008
	L_t	0.7649	0.9012	0.9689	0.9974	1.0029	0.9947	0.9784
	T_t	0.0012	0.0004	0.0000	0.0003	0.0013	0.0030	0.0054
10°	L_r	0.2108	0.0884	0.0286	0.0042	0.0004	0.0087	0.0241
	T_r	0.0203	0.0082	0.0025	0.0003	0.0001	0.0011	0.0027
	L_t	0.7552	0.8956	0.9688	1.0031	1.0141	1.0110	0.9993
	T_t	0.0047	0.0015	0.0000	0.0012	0.0052	0.0122	0.0217
15°	L_r	0.1868	0.0787	0.0257	0.0040	0.0003	0.0074	0.0208
	T_r	0.0426	0.0173	0.0053	0.0007	0.0001	0.0018	0.0045
	L_t	0.7392	0.8863	0.9688	1.0133	1.0350	1.0432	1.0442
	T_t	0.0105	0.0034	0.0000	0.0027	0.0120	0.0280	0.0504
20°	L_r	0.1574	0.0666	0.0220	0.0035	0.0003	0.0064	0.0191
	T_r	0.0685	0.0279	0.0088	0.0013	0.0001	0.0018	0.0043
	L_t	0.7170	0.8730	0.9688	1.0292	1.0702	1.1031	1.1397
	T_t	0.0183	0.0059	0.0000	0.0048	0.0217	0.0514	0.0937
25°	L_r	0.1256	0.0536	0.0178	0.0027	0.0003	0.0074	0.0269
	T_r	0.0943	0.0384	0.0125	0.0023	0.0000	0.0007	0.0008
	L_t	0.6890	0.8557	0.9687	1.0532	1.1292	1.2220	1.3899
	T_t	0.0278	0.0091	0.0001	0.0077	0.0351	0.0845	0.1586
30°	L_r	0.0948	0.0410	0.0136	0.0017	0.0011	0.0182	0.5769
	T_r	0.1159	0.0472	0.0160	0.0038	0.0005	0.0006	0.1336
	L_t	0.6556	0.8338	0.9687	1.0893	1.2344	1.5166	4.6944
	T_t	0.0387	0.0127	0.0001	0.0113	0.0532	0.1338	0.2922

$\bar{\gamma}$ = (density of refracting medium)/(density of incident medium) = γ_2/γ_1.
$\bar{\alpha}$ = (velocity in refracting medium)/(velocity in incident medium) = α_2/α_1.
L_r, T_r, L_t, T_t = intensities of reflected longitudinal, reflected transverse, transmitted longitudinal, and transmitted transverse waves per unit intensity of incident longitudinal waves; θ = angle of incidence.

TABLE 2—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=0.8$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	L_r 0.1837	0.0625	0.0123	0.0000	0.0083	0.0278	0.0533
	T_r 0	0	0	0	0	0	0
	L_t 0.8163	0.9375	0.9877	1.0000	0.9917	0.9723	0.9467
	T_t 0	0	0	0	0	0	0
5°	L_r 0.1793	0.0611	0.0121	0.0000	0.0080	0.0269	0.0516
	T_r 0.0042	0.0014	0.0003	0.0000	0.0002	0.0007	0.0013
	L_t 0.8129	0.9356	0.9877	1.0018	0.9952	0.9771	0.9527
	T_t 0.0012	0.0004	0.0000	0.0003	0.0014	0.0031	0.0053
10°	L_r 0.1669	0.0569	0.0114	0.0000	0.0072	0.0245	0.0471
	T_r 0.0161	0.0053	0.0010	0.0000	0.0008	0.0026	0.0048
	L_t 0.8026	0.9299	0.9876	1.0076	1.0062	0.9930	0.9730
	T_t 0.0049	0.0015	0.0000	0.0013	0.0054	0.0123	0.0217
15°	L_r 0.1478	0.0506	0.0102	0.0000	0.0062	0.0213	0.0412
	T_r 0.0337	0.0112	0.0021	0.0000	0.0015	0.0047	0.0086
	L_t 0.7856	0.9203	0.9876	1.0178	1.0271	1.0247	1.0167
	T_t 0.0109	0.0033	0.0000	0.0029	0.0123	0.0282	0.0501
20°	L_r 0.1245	0.0428	0.0087	0.0000	0.0052	0.0185	0.0372
	T_r 0.0541	0.0179	0.0035	0.0000	0.0018	0.0058	0.0099
	L_t 0.7621	0.9065	0.9876	1.0339	1.0622	1.0839	1.1108
	T_t 0.0190	0.0058	0.0000	0.0052	0.0223	0.0514	0.0922
25°	L_r 0.0994	0.0345	0.0071	0.0000	0.0048	0.0186	0.0452
	T_r 0.0740	0.0244	0.0049	0.0000	0.0016	0.0045	0.0048
	L_t 0.7323	0.8884	0.9876	1.0582	1.1213	1.2027	1.3606
	T_t 0.0288	0.0089	0.0000	0.0082	0.0357	0.0836	0.1531
30°	L_r 0.0752	0.0265	0.0054	0.0000	0.0061	0.0318	0.6542
	T_r 0.0905	0.0297	0.0063	0.0002	0.0006	0.0005	0.1110
	L_t 0.6968	0.8657	0.9876	1.0948	1.2277	1.5010	4.8300
	T_t 0.0400	0.0125	0.0000	0.0120	0.0533	0.1290	0.2364
	$\bar{\gamma}=\gamma_2/\gamma_1=0.9$						
0°	L_r 0.1439	0.0376	0.0028	0.0035	0.0222	0.0499	0.0816
	T_r 0	0	0	0	0	0	0
	L_t 0.8561	0.9624	0.9972	0.9964	0.9778	0.9501	0.9183
	T_t 0	0	0	0	0	0	0
5°	L_r 0.1404	0.0368	0.0027	0.0034	0.0216	0.0485	0.0793
	T_r 0.0033	0.0008	0.0001	0.0001	0.0005	0.0012	0.0019
	L_t 0.8525	0.9604	0.9972	0.9983	0.9812	0.9549	0.9241
	T_t 0.0013	0.0004	0.0000	0.0003	0.0014	0.0031	0.0053
10°	L_r 0.1306	0.0342	0.0026	0.0031	0.0198	0.0446	0.0729
	T_r 0.0126	0.0032	0.0002	0.0003	0.0020	0.0044	0.0070
	L_t 0.8418	0.9546	0.9972	1.0040	0.9921	0.9703	0.9438
	T_t 0.0050	0.0015	0.0000	0.0014	0.0056	0.0124	0.0215
15°	L_r 0.1156	0.0304	0.0023	0.0026	0.0173	0.0391	0.0644
	T_r 0.0264	0.0067	0.0005	0.0006	0.0038	0.0083	0.0132
	L_t 0.8240	0.9445	0.9972	1.0143	1.0126	1.0012	0.9862
	T_t 0.0112	0.0033	0.0000	0.0031	0.0126	0.0282	0.0494
20°	L_r 0.0972	0.0256	0.0020	0.0022	0.0143	0.0340	0.0575
	T_r 0.0422	0.0107	0.0007	0.0009	0.0053	0.0112	0.0164
	L_t 0.7993	0.9306	0.9973	1.0302	1.0477	1.0596	1.0784
	T_t 0.0195	0.0057	0.0000	0.0055	0.0229	0.0510	0.0903
25°	L_r 0.0777	0.0206	0.0016	0.0019	0.0130	0.0324	0.0646
	T_r 0.0575	0.0144	0.0011	0.0009	0.0057	0.0106	0.0109
	L_t 0.7682	0.9121	0.9972	1.0547	1.1062	1.1774	1.3259
	T_t 0.0296	0.0087	0.0000	0.0086	0.0359	0.0822	0.1473
30°	L_r 0.0589	0.0160	0.0012	0.0018	0.0137	0.0465	0.7189
	T_r 0.0698	0.0173	0.0014	0.0008	0.0043	0.0040	0.0920
	L_t 0.7308	0.8886	0.9972	1.0916	1.2131	1.4769	4.9317
	T_t 0.0410	0.0121	0.0000	0.0125	0.0529	0.1240	0.1902

TABLE 2—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.0$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	L_r 0.1111	0.0204	0	0.0123	0.0400	0.0744	0.1111
	T_r 0	0	0	0	0	0	0
	L_t 0.8889	0.9796	1.0000	0.9876	0.9600	0.9257	0.8889
	T_t 0	0	0	0	0	0	0
5°	L_r 0.1084	0.0199	0	0.0120	0.0390	0.0725	0.1081
	T_r 0.0026	0.0005	0	0.0003	0.0009	0.0017	0.0025
	L_t 0.8852	0.9776	1.0000	0.9894	0.9632	0.9302	0.8944
	T_t 0.0013	0.0004	0	0.0004	0.0014	0.0031	0.0052
10°	L_r 0.1007	0.0185	0	0.0112	0.0360	0.0669	0.0999
	T_r 0.0098	0.0018	0	0.0011	0.0034	0.0063	0.0093
	L_t 0.8740	0.9717	1.0000	0.9951	0.9740	0.9452	0.9133
	T_t 0.0052	0.0014	0	0.0014	0.0056	0.0123	0.0211
15°	L_r 0.0890	0.0163	0	0.0098	0.0318	0.0592	0.0886
	T_r 0.0204	0.0037	0	0.0021	0.0068	0.0123	0.0178
	L_t 0.8556	0.9617	1.0000	1.0051	0.9941	0.9754	0.9546
	T_t 0.0114	0.0032	0	0.0033	0.0128	0.0280	0.0484
20°	L_r 0.0748	0.0137	0	0.0083	0.0273	0.0513	0.0788
	T_r 0.0325	0.0058	0	0.0032	0.0099	0.0173	0.0234
	L_t 0.8300	0.9474	1.0000	1.0212	1.0282	1.0325	1.0444
	T_t 0.0198	0.0055	0	0.0058	0.0228	0.0505	0.0881
25°	L_r 0.0597	0.0111	0	0.0069	0.0235	0.0475	0.0843
	T_r 0.0440	0.0076	0	0.0040	0.0116	0.0179	0.0183
	L_t 0.7977	0.9284	1.0000	1.0453	1.0866	1.1488	1.2885
	T_t 0.0300	0.0084	0	0.0089	0.0359	0.0805	0.1416
30°	L_r 0.0454	0.0086	0	0.0060	0.0229	0.0616	0.7728
	T_r 0.0530	0.0090	0	0.0042	0.0104	0.0098	0.0759
	L_t 0.7589	0.9045	1.0000	1.0824	1.1930	1.4474	5.0115
	T_t 0.0416	0.0118	0	0.0128	0.0523	0.1190	0.1519
			$\bar{\gamma}=\gamma_2/\gamma_1=1.1$				
0°	L_r 0.0843	0.0092	0.0023	0.0249	0.0602	0.1001	0.1406
	T_r 0	0	0	0	0	0	0
	L_t 0.9157	0.9908	0.9977	0.9751	0.9398	0.8998	0.8594
	T_t 0	0	0	0	0	0	0
5°	L_r 0.0822	0.0090	0.0022	0.0243	0.0587	0.0975	0.1370
	T_r 0.0020	0.0002	0.0000	0.0005	0.0013	0.0022	0.0031
	L_t 0.9119	0.9888	0.9977	0.9769	0.9429	0.9044	0.8647
	T_t 0.0013	0.0004	0.0000	0.0004	0.0014	0.0030	0.0051
10°	L_r 0.0763	0.0083	0.0021	0.0227	0.0545	0.0904	0.1270
	T_r 0.0074	0.0008	0.0002	0.0021	0.0050	0.0083	0.0115
	L_t 0.9004	0.9828	0.9978	0.9824	0.9534	0.9190	0.8829
	T_t 0.0052	0.0014	0.0000	0.0015	0.0057	0.0122	0.0207
15°	L_r 0.0673	0.0072	0.0019	0.0201	0.0484	0.0803	0.1130
	T_r 0.0156	0.0017	0.0004	0.0042	0.0101	0.0164	0.0224
	L_t 0.8814	0.9727	0.9977	0.9923	0.9730	0.9483	0.9228
	T_t 0.0116	0.0031	0.0000	0.0034	0.0128	0.0277	0.0474
20°	L_r 0.0567	0.0060	0.0016	0.0172	0.0415	0.0696	0.1002
	T_r 0.0247	0.0026	0.0006	0.0066	0.0151	0.0237	0.0305
	L_t 0.8549	0.9582	0.9977	1.0081	1.0066	1.0041	1.0102
	T_t 0.0200	0.0053	0.0000	0.0059	0.0228	0.0497	0.0857
25°	L_r 0.0450	0.0048	0.0013	0.0142	0.0355	0.0633	0.1036
	T_r 0.0331	0.0033	0.0009	0.0085	0.0184	0.0260	0.0263
	L_t 0.8218	0.9391	0.9978	1.0321	1.0642	1.1184	1.2504
	T_t 0.0303	0.0082	0.0000	0.0092	0.0357	0.0786	0.1360
30°	L_r 0.0343	0.0038	0.0010	0.0119	0.0331	0.0765	0.8176
	T_r 0.0395	0.0037	0.0011	0.0095	0.0180	0.0171	0.0623
	L_t 0.7819	0.9148	0.9978	1.0689	1.1698	1.4149	5.0695
	T_t 0.0419	0.0114	0.0000	0.0131	0.0514	0.1142	0.1204

TABLE 2—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.2$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	L_r 0.0625	0.0028	0.0083	0.0400	0.0816	0.1259	0.1696
	T_r 0	0	0	0	0	0	0
	L_t 0.9375	0.9972	0.9917	0.9600	0.9183	0.8742	0.8305
	T_t 0	0	0	0	0	0	0
5°	L_r 0.0609	0.0027	0.0081	0.0391	0.0797	0.1229	0.1654
	T_r 0.0015	0.0001	0.0002	0.0009	0.0018	0.0027	0.0037
	L_t 0.9336	0.9952	0.9916	0.9617	0.9214	0.8784	0.8354
	T_t 0.0013	0.0003	0.0000	0.0004	0.0014	0.0030	0.0050
10°	L_r 0.0564	0.0024	0.0076	0.0365	0.0742	0.1142	0.1536
	T_r 0.0056	0.0003	0.0006	0.0032	0.0066	0.0102	0.0137
	L_t 0.9219	0.9892	0.9918	0.9673	0.9316	0.8924	0.8531
	T_t 0.0052	0.0013	0.0000	0.0016	0.0057	0.0121	0.0203
15°	L_r 0.0497	0.0021	0.0069	0.0326	0.0662	0.1018	0.1370
	T_r 0.0116	0.0006	0.0014	0.0067	0.0136	0.0205	0.0270
	L_t 0.9025	0.9791	0.9917	0.9770	0.9507	0.9209	0.8917
	T_t 0.0116	0.0030	0.0000	0.0035	0.0128	0.0274	0.0463
20°	L_r 0.0416	0.0017	0.0059	0.0279	0.0568	0.0883	0.1212
	T_r 0.0182	0.0008	0.0023	0.0106	0.0207	0.0302	0.0376
	L_t 0.8756	0.9645	0.9918	0.9925	0.9837	0.9753	0.9766
	T_t 0.0201	0.0052	0.0000	0.0061	0.0227	0.0489	0.0833
25°	L_r 0.0331	0.0014	0.0048	0.0231	0.0483	0.0792	0.1222
	T_r 0.0243	0.0009	0.0032	0.0140	0.0259	0.0344	0.0347
	L_t 0.8415	0.9452	0.9917	1.0163	1.0402	1.0874	1.2122
	T_t 0.0305	0.0079	0.0000	0.0094	0.0354	0.0768	0.1307
30°	L_r 0.0253	0.0011	0.0037	0.0189	0.0437	0.0909	0.8547
	T_r 0.0286	0.0009	0.0041	0.0162	0.0266	0.0254	0.0507
	L_t 0.8005	0.9207	0.9918	1.0528	1.1447	1.3809	5.1120
	T_t 0.0421	0.0118	0.0000	0.0132	0.0506	0.1096	0.0947
			$\bar{\gamma}=\gamma_2/\gamma_1=1.3$				
0°	L_r 0.0450	0.0002	0.0170	0.0567	0.1037	0.1516	0.1975
	T_r 0	0	0	0	0	0	0
	L_t 0.9550	0.9998	0.9830	0.9433	0.8963	0.8485	0.8025
	T_t 0	0	0	0	0	0	0
5°	L_r 0.0438	0.0002	0.0167	0.0555	0.1013	0.1480	0.1927
	T_r 0.0011	0.0000	0.0003	0.0012	0.0022	0.0032	0.0042
	L_t 0.9510	0.9978	0.9831	0.9449	0.8994	0.8525	0.8074
	T_t 0.0013	0.0003	0.0000	0.0004	0.0014	0.0030	0.0049
10°	L_r 0.0405	0.0001	0.0157	0.0519	0.0946	0.1379	0.1794
	T_r 0.0041	0.0000	0.0013	0.0045	0.0083	0.0121	0.0157
	L_t 0.9391	0.9918	0.9830	0.9504	0.9092	0.8662	0.8243
	T_t 0.0053	0.0013	0.0000	0.0016	0.0057	0.0119	0.0198
15°	L_r 0.0356	0.0001	0.0142	0.0464	0.0844	0.1231	0.1604
	T_r 0.0084	0.0000	0.0028	0.0094	0.0171	0.0246	0.0313
	L_t 0.9194	0.9816	0.9830	0.9599	0.9278	0.8938	0.8616
	T_t 0.0116	0.0028	0.0000	0.0036	0.0128	0.0269	0.0452
20°	L_r 0.0297	0.0001	0.0122	0.0398	0.0727	0.1069	0.1416
	T_r 0.0131	0.0000	0.0046	0.0150	0.0264	0.0366	0.0444
	L_t 0.8920	0.9670	0.9830	0.9752	0.9599	0.9468	0.9441
	T_t 0.0202	0.0050	0.0000	0.0062	0.0226	0.0479	0.0810
25°	L_r 0.0236	0.0000	0.0100	0.0329	0.0615	0.0949	0.1400
	T_r 0.0173	0.0000	0.0066	0.0201	0.0336	0.0430	0.0432
	L_t 0.8573	0.9477	0.9830	0.9986	1.0156	1.0565	1.1749
	T_t 0.0305	0.0076	0.0000	0.0095	0.0350	0.0747	0.1257
30°	L_r 0.0181	0.0000	0.0077	0.0267	0.0546	0.1049	0.8853
	T_r 0.0200	0.0000	0.0085	0.0237	0.0357	0.0342	0.0409
	L_t 0.8155	0.9231	0.9830	1.0347	1.1185	1.3463	5.1412
	T_t 0.0420	0.0105	0.0000	0.0133	0.0496	0.1051	0.0738

TABLE 3

FRACTIONS OF THE INCIDENT ENERGY TRANSFORMED INTO THE LONGITUDINAL AND TRANSVERSE WAVES CREATED AT AN ELASTIC INTERFACE AFTER INCIDENCE OF A LONGITUDINAL WAVE

$\bar{\gamma}=\gamma_2/\gamma_1=0.7$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	$\bar{L}_r$ 0.2318	0.0970	0.0311	0.0044	0.0006	0.0102	0.0278
	$\bar{T}_r$ 0	0	0	0	0	0	0
	$\bar{L}_t$ 0.7682	0.9030	0.9689	0.9956	0.9994	0.9899	0.9722
	$\bar{T}_t$ 0	0	0	0	0	0	0
5°	$\bar{L}_r$ 0.2264	0.0948	0.0305	0.0044	0.0006	0.0098	0.0268
	$\bar{T}_r$ 0.0053	0.0021	0.0006	0.0001	0.0000	0.0003	0.0008
	$\bar{L}_t$ 0.7671	0.9027	0.9689	0.9952	0.9981	0.9869	0.9671
	$\bar{T}_t$ 0.0012	0.0004	0.0000	0.0003	0.0013	0.0030	0.0053
10°	$\bar{L}_r$ 0.2108	0.0884	0.0286	0.0042	0.0004	0.0087	0.0241
	$\bar{T}_r$ 0.0205	0.0083	0.0025	0.0003	0.0001	0.0011	0.0027
	$\bar{L}_t$ 0.7640	0.9017	0.9688	0.9943	0.9942	0.9780	0.9516
	$\bar{T}_t$ 0.0048	0.0015	0.0000	0.0012	0.0053	0.0122	0.0216
15°	$\bar{L}_r$ 0.1868	0.0787	0.0257	0.0040	0.0003	0.0074	0.0208
	$\bar{T}_r$ 0.0436	0.0176	0.0054	0.0007	0.0001	0.0017	0.0044
	$\bar{L}_t$ 0.7588	0.9001	0.9688	0.9926	0.9875	0.9629	0.9249
	$\bar{T}_t$ 0.0108	0.0035	0.0000	0.0027	0.0121	0.0280	0.0498
20°	$\bar{L}_r$ 0.1574	0.0666	0.0220	0.0035	0.0003	0.0064	0.0191
	$\bar{T}_r$ 0.0715	0.0291	0.0092	0.0014	0.0001	0.0019	0.0045
	$\bar{L}_t$ 0.7518	0.8979	0.9688	0.9901	0.9776	0.9404	0.8847
	$\bar{T}_t$ 0.0194	0.0063	0.0000	0.0050	0.0220	0.0513	0.0916
25°	$\bar{L}_r$ 0.1256	0.0536	0.0178	0.0027	0.0003	0.0074	0.0269
	$\bar{T}_r$ 0.1009	0.0411	0.0134	0.0025	0.0000	0.0008	0.0009
	$\bar{L}_t$ 0.7431	0.8954	0.9687	0.9867	0.9636	0.9075	0.8195
	$\bar{T}_t$ 0.0304	0.0099	0.0001	0.0081	0.0360	0.0843	0.1527
30°	$\bar{L}_r$ 0.0948	0.0410	0.0136	0.0017	0.0011	0.0182	0.5769
	$\bar{T}_r$ 0.1281	0.0522	0.0177	0.0042	0.0006	0.0007	0.1477
	$\bar{L}_t$ 0.7330	0.8925	0.9687	0.9819	0.9428	0.8478	0
	$\bar{T}_t$ 0.0442	0.0143	0.0000	0.0122	0.0554	0.1333	0.2755

$\bar{\gamma}$ = (density of refracting medium)/(density of incident medium) = γ_2/γ_1.
$\bar{\alpha}$ = (velocity in refracting medium)/(velocity in incident medium) = α_2/α_1.
$\bar{L}_r$, $\bar{T}_r$, $\bar{L}_t$, $\bar{T}_t$ = fractions of incident energy transformed into reflected longitudinal, reflected transverse, transmitted longitudinal, and transmitted transverse waves; θ = angle of incidence.

TABLE 3—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=0.8$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	$\bar{L}_r$ 0.1837	0.0625	0.0123	0.0000	0.0083	0.0278	0.0533
	$\bar{T}_r$ 0	0	0	0	0	0	0
	$\bar{L}_t$ 0.8163	0.9375	0.9877	1.0000	0.9917	0.9723	0.9467
	$\bar{T}_t$ 0	0	0	0	0	0	0
5°	$\bar{L}_r$ 0.1793	0.0611	0.0121	0.0000	0.0080	0.0269	0.0516
	$\bar{T}_r$ 0.0042	0.0014	0.0003	0.0000	0.0002	0.0007	0.0013
	$\bar{L}_t$ 0.8152	0.9372	0.9877	0.9997	0.9904	0.9694	0.9417
	$\bar{T}_t$ 0.0013	0.0004	0.0000	0.0003	0.0014	0.0031	0.0053
10°	$\bar{L}_r$ 0.1669	0.0569	0.0114	0.0000	0.0072	0.0245	0.0471
	$\bar{T}_r$ 0.0163	0.0054	0.0010	0.0000	0.0008	0.0026	0.0048
	$\bar{L}_t$ 0.8119	0.9362	0.9876	0.9987	0.9865	0.9606	0.9265
	$\bar{T}_t$ 0.0050	0.0015	0.0000	0.0013	0.0054	0.0123	0.0216
15°	$\bar{L}_r$ 0.1478	0.0506	0.0102	0.0000	0.0062	0.0213	0.0412
	$\bar{T}_r$ 0.0345	0.0115	0.0021	0.0000	0.0015	0.0048	0.0088
	$\bar{L}_t$ 0.8065	0.9346	0.9877	0.9970	0.9799	0.9458	0.9006
	$\bar{T}_t$ 0.0113	0.0034	0.0000	0.0030	0.0124	0.0282	0.0495
20°	$\bar{L}_r$ 0.1245	0.0428	0.0087	0.0000	0.0052	0.0185	0.0372
	$\bar{T}_r$ 0.0564	0.0187	0.0037	0.0000	0.0019	0.0061	0.0103
	$\bar{L}_t$ 0.7991	0.9324	0.9876	0.9946	0.9703	0.9240	0.8623
	$\bar{T}_t$ 0.0201	0.0061	0.0000	0.0054	0.0227	0.0513	0.0901
25°	$\bar{L}_r$ 0.0994	0.0345	0.0071	0.0000	0.0048	0.0186	0.0452
	$\bar{T}_r$ 0.0792	0.0261	0.0052	0.0000	0.0017	0.0048	0.0051
	$\bar{L}_t$ 0.7898	0.9297	0.9876	0.9914	0.9568	0.8932	0.8023
	$\bar{T}_t$ 0.0316	0.0097	0.0000	0.0086	0.0367	0.0834	0.1474
30°	$\bar{L}_r$ 0.0752	0.0265	0.0054	0.0000	0.0061	0.0318	0.6542
	$\bar{T}_r$ 0.1001	0.0328	0.0070	0.0002	0.0007	0.0006	0.1227
	$\bar{L}_t$ 0.7790	0.9267	0.9876	0.9868	0.9377	0.8391	0
	$\bar{T}_t$ 0.0457	0.0141	0.0000	0.0129	0.0555	0.1285	0.2230

$\bar{\gamma}=\gamma_2/\gamma_1=0.9$

0°	$\bar{L}_r$ 0.1439	0.0376	0.0028	0.0035	0.0222	0.0499	0.0816
	$\bar{T}_r$ 0	0	0	0	0	0	0
	$\bar{L}_t$ 0.8561	0.9624	0.9972	0.9964	0.9778	0.9501	0.9183
	$\bar{T}_t$ 0	0	0	0	0	0	0
5°	$\bar{L}_r$ 0.1404	0.0368	0.0027	0.0034	0.0216	0.0485	0.0793
	$\bar{T}_r$ 0.0033	0.0008	0.0001	0.0001	0.0005	0.0012	0.0019
	$\bar{L}_t$ 0.8550	0.9620	0.9972	0.9961	0.9765	0.9473	0.9135
	$\bar{T}_t$ 0.0013	0.0004	0.0000	0.0003	0.0014	0.0031	0.0053
10°	$\bar{L}_r$ 0.1306	0.0342	0.0026	0.0031	0.0198	0.0446	0.0729
	$\bar{T}_r$ 0.0128	0.0034	0.0002	0.0003	0.0020	0.0044	0.0071
	$\bar{L}_t$ 0.8515	0.9610	0.9972	0.9952	0.9727	0.9387	0.8987
	$\bar{T}_t$ 0.0051	0.0015	0.0000	0.0014	0.0056	0.0124	0.0214
15°	$\bar{L}_r$ 0.1156	0.0304	0.0023	0.0026	0.0173	0.0391	0.0643
	$\bar{T}_r$ 0.0271	0.0069	0.0005	0.0006	0.0039	0.0085	0.0135
	$\bar{L}_t$ 0.8458	0.9593	0.9972	0.9936	0.9661	0.9241	0.8735
	$\bar{T}_t$ 0.0116	0.0034	0.0000	0.0031	0.0127	0.0282	0.0488
20°	$\bar{L}_r$ 0.0972	0.0256	0.0020	0.0022	0.0143	0.0340	0.0575
	$\bar{T}_r$ 0.0440	0.0112	0.0007	0.0009	0.0055	0.0117	0.0171
	$\bar{L}_t$ 0.8381	0.9572	0.9973	0.9911	0.9570	0.9033	0.8371
	$\bar{T}_t$ 0.0206	0.0060	0.0000	0.0057	0.0233	0.0509	0.0883
25°	$\bar{L}_r$ 0.0777	0.0206	0.0016	0.0019	0.0130	0.0324	0.0646
	$\bar{T}_r$ 0.0615	0.0155	0.0012	0.0010	0.0061	0.0113	0.0117
	$\bar{L}_t$ 0.8284	0.9545	0.9973	0.9881	0.9439	0.8744	0.7818
	$\bar{T}_t$ 0.0324	0.0094	0.0000	0.0091	0.0369	0.0820	0.1418
30°	$\bar{L}_r$ 0.0589	0.0160	0.0012	0.0018	0.0137	0.0465	0.7189
	$\bar{T}_r$ 0.0772	0.0191	0.0016	0.0008	0.0047	0.0044	0.1018
	$\bar{L}_t$ 0.8171	0.9512	0.9972	0.9839	0.9265	0.8256	0
	$\bar{T}_t$ 0.0468	0.0137	0.0000	0.0134	0.0551	0.1236	0.1793

TABLE 3—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.0$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	$\bar{L}_r$ 0.1111	0.0204	0	0.0123	0.0400	0.0744	0.1111
	$\bar{T}_r$ 0	0	0	0	0	0	0
	$\bar{L}_t$ 0.8889	0.9796	1.0000	0.9876	0.9600	0.9257	0.8889
	$\bar{T}_t$ 0	0	0	0	0	0	0
5°	$\bar{L}_r$ 0.1084	0.0199	0	0.0120	0.0390	0.0725	0.1081
	$\bar{T}_r$ 0.0026	0.0005	0	0.0003	0.0009	0.0017	0.0026
	$\bar{L}_t$ 0.8877	0.9793	1.0000	0.9873	0.9586	0.9228	0.8841
	$\bar{T}_t$ 0.0013	0.0004	0	0.0004	0.0014	0.0031	0.0052
10°	$\bar{L}_r$ 0.1007	0.0185	0	0.0112	0.0360	0.0669	0.0999
	$\bar{T}_r$ 0.0099	0.0018	0	0.0011	0.0034	0.0064	0.0094
	$\bar{L}_t$ 0.8841	0.9783	1.0000	0.9864	0.9549	0.9144	0.8697
	$\bar{T}_t$ 0.0052	0.0014	0	0.0014	0.0057	0.0123	0.0210
15°	$\bar{L}_r$ 0.0890	0.0163	0	0.0098	0.0318	0.0592	0.0886
	$\bar{T}_r$ 0.0209	0.0038	0	0.0022	0.0070	0.0126	0.0182
	$\bar{L}_t$ 0.8783	0.9767	1.0000	0.9846	0.9484	0.9003	0.8455
	$\bar{T}_t$ 0.0118	0.0033	0	0.0034	0.0129	0.0280	0.0478
20°	$\bar{L}_r$ 0.0748	0.0137	0	0.0083	0.0273	0.0513	0.0788
	$\bar{T}_r$ 0.0339	0.0060	0	0.0033	0.0103	0.0181	0.0244
	$\bar{L}_t$ 0.8703	0.9744	1.0000	0.9824	0.9392	0.8802	0.8108
	$\bar{T}_t$ 0.0210	0.0058	0	0.0060	0.0232	0.0504	0.0861
25°	$\bar{L}_r$ 0.0597	0.0111	0	0.0069	0.0235	0.0475	0.0843
	$\bar{T}_r$ 0.0471	0.0082	0	0.0043	0.0124	0.0191	0.0196
	$\bar{L}_t$ 0.8603	0.9716	1.0000	0.9793	0.9272	0.8531	0.7597
	$\bar{T}_t$ 0.0329	0.0091	0	0.0094	0.0369	0.0803	0.1364
30°	$\bar{L}_r$ 0.0454	0.0086	0	0.0060	0.0229	0.0616	0.7728
	$\bar{T}_r$ 0.0586	0.0099	0	0.0046	0.0115	0.0108	0.0840
	$\bar{L}_t$ 0.8484	0.9682	1.0000	0.9756	0.9112	0.8091	0
	$\bar{T}_t$ 0.0475	0.0133	0	0.0138	0.0544	0.1185	0.1432

$\bar{\gamma}=\gamma_2/\gamma_1=1.1$

0°	$\bar{L}_r$ 0.0843	0.0092	0.0023	0.0249	0.0602	0.1001	0.1406
	$\bar{T}_r$ 0	0	0	0	0	0	0
	$\bar{L}_t$ 0.9157	0.9908	0.9977	0.9751	0.9398	0.8998	0.8594
	$\bar{T}_t$ 0	0	0	0	0	0	0
5°	$\bar{L}_r$ 0.0822	0.0090	0.0022	0.0243	0.0587	0.0975	0.1370
	$\bar{T}_r$ 0.0020	0.0002	0.0001	0.0005	0.0013	0.0022	0.0031
	$\bar{L}_t$ 0.9145	0.9905	0.9977	0.9748	0.9386	0.8972	0.8547
	$\bar{T}_t$ 0.0013	0.0004	0.0000	0.0004	0.0014	0.0030	0.0051
10°	$\bar{L}_r$ 0.0763	0.0083	0.0021	0.0227	0.0545	0.0904	0.1270
	$\bar{T}_r$ 0.0075	0.0008	0.0002	0.0021	0.0051	0.0084	0.0117
	$\bar{L}_t$ 0.9109	0.9895	0.9977	0.9738	0.9347	0.8890	0.8407
	$\bar{T}_t$ 0.0053	0.0014	0.0000	0.0015	0.0057	0.0122	0.0206
15°	$\bar{L}_r$ 0.0673	0.0072	0.0019	0.0201	0.0484	0.0803	0.1130
	$\bar{T}_r$ 0.0159	0.0017	0.0004	0.0043	0.0103	0.0168	0.0229
	$\bar{L}_t$ 0.9049	0.9879	0.9977	0.9721	0.9283	0.8753	0.8174
	$\bar{T}_t$ 0.0119	0.0032	0.0000	0.0035	0.0129	0.0277	0.0468
20°	$\bar{L}_r$ 0.0567	0.0060	0.0016	0.0172	0.0415	0.0696	0.1002
	$\bar{T}_r$ 0.0258	0.0027	0.0006	0.0069	0.0158	0.0247	0.0318
	$\bar{L}_t$ 0.8963	0.9856	0.9977	0.9698	0.9195	0.8560	0.7842
	$\bar{T}_t$ 0.0212	0.0056	0.0000	0.0061	0.0232	0.0496	0.0838
25°	$\bar{L}_r$ 0.0450	0.0048	0.0013	0.0142	0.0355	0.0632	0.1036
	$\bar{T}_r$ 0.0354	0.0036	0.0010	0.0091	0.0197	0.0278	0.0281
	$\bar{L}_t$ 0.8863	0.9828	0.9978	0.9670	0.9081	0.8306	0.7373
	$\bar{T}_t$ 0.0332	0.0088	0.0000	0.0097	0.0367	0.0784	0.1310
30°	$\bar{L}_r$ 0.0343	0.0038	0.0010	0.0119	0.0331	0.0764	0.8176
	$\bar{T}_r$ 0.0436	0.0041	0.0012	0.0105	0.0199	0.0189	0.0689
	$\bar{L}_t$ 0.8742	0.9793	0.9978	0.9635	0.8935	0.7909	0
	$\bar{T}_t$ 0.0479	0.0128	0.0000	0.0141	0.0535	0.1138	0.1135

TABLE 3—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.2$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	$\bar{L}_r$ 0.0625	0.0028	0.0083	0.0400	0.0816	0.1259	0.1696
	$\bar{T}_r$ 0	0	0	0	0	0	0
	$\bar{L}_t$ 0.9375	0.9972	0.9917	0.9600	0.9183	0.8742	0.8305
	$\bar{T}_t$ 0	0	0	0	0	0	0
5°	$\bar{L}_r$ 0.0609	0.0027	0.0081	0.0391	0.0797	0.1229	0.1654
	$\bar{T}_r$ 0.0015	0.0001	0.0002	0.0009	0.0018	0.0027	0.0037
	$\bar{L}_t$ 0.9363	0.9969	0.9916	0.9596	0.9170	0.8714	0.8258
	$\bar{T}_t$ 0.0013	0.0003	0.0000	0.0004	0.0014	0.0030	0.0050
10°	$\bar{L}_r$ 0.0564	0.0024	0.0076	0.0365	0.0742	0.1142	0.1536
	$\bar{T}_r$ 0.0057	0.0003	0.0006	0.0032	0.0067	0.0103	0.0138
	$\bar{L}_t$ 0.9326	0.9960	0.9918	0.9588	0.9134	0.8633	0.8123
	$\bar{T}_t$ 0.0053	0.0014	0.0000	0.0016	0.0057	0.0121	0.0202
15°	$\bar{L}_r$ 0.0497	0.0021	0.0069	0.0326	0.0662	0.1018	0.1370
	$\bar{T}_r$ 0.0119	0.0006	0.0014	0.0069	0.0139	0.0210	0.0276
	$\bar{L}_t$ 0.9265	0.9943	0.9917	0.9570	0.9070	0.8500	0.7898
	$\bar{T}_t$ 0.0120	0.0030	0.0000	0.0036	0.0129	0.0273	0.0457
20°	$\bar{L}_r$ 0.0416	0.0017	0.0059	0.0279	0.0568	0.0883	0.1212
	$\bar{T}_r$ 0.0190	0.0008	0.0024	0.0111	0.0216	0.0315	0.0392
	$\bar{L}_t$ 0.9181	0.9920	0.9918	0.9548	0.8986	0.8314	0.7581
	$\bar{T}_t$ 0.0213	0.0054	0.0000	0.0063	0.0231	0.0488	0.0814
25°	$\bar{L}_r$ 0.0331	0.0014	0.0048	0.0231	0.0483	0.0792	0.1222
	$\bar{T}_r$ 0.0260	0.0010	0.0034	0.0150	0.0277	0.0368	0.0371
	$\bar{L}_t$ 0.9075	0.9891	0.9917	0.9521	0.8876	0.8075	0.7147
	$\bar{T}_t$ 0.0334	0.0085	0.0000	0.0099	0.0363	0.0766	0.1259
30°	$\bar{L}_r$ 0.0253	0.0011	0.0037	0.0189	0.0437	0.0909	0.8547
	$\bar{T}_r$ 0.0316	0.0010	0.0045	0.0179	0.0294	0.0280	0.0560
	$\bar{L}_t$ 0.8950	0.9856	0.9918	0.9490	0.8743	0.7719	0
	$\bar{T}_t$ 0.0480	0.0124	0.0000	0.0142	0.0527	0.1092	0.0893

$\bar{\gamma}=\gamma_2/\gamma_1=1.3$

0°	$\bar{L}_r$ 0.0450	0.0002	0.0170	0.0567	0.1037	0.1516	0.1975
	$\bar{T}_r$ 0	0	0	0	0	0	0
	$\bar{L}_t$ 0.9550	0.9998	0.9830	0.9433	0.8963	0.8485	0.8025
	$\bar{T}_t$ 0	0	0	0	0	0	0
5°	$\bar{L}_r$ 0.0438	0.0002	0.0167	0.0555	0.1013	0.1480	0.1927
	$\bar{T}_r$ 0.0011	0.0000	0.0003	0.0012	0.0022	0.0032	0.0042
	$\bar{L}_t$ 0.9537	0.9995	0.9831	0.9429	0.8951	0.8458	0.7981
	$\bar{T}_t$ 0.0013	0.0003	0	0.0004	0.0014	0.0030	0.0049
10°	$\bar{L}_r$ 0.0405	0.0001	0.0157	0.0519	0.0946	0.1379	0.1794
	$\bar{T}_r$ 0.0042	0.0000	0.0013	0.0045	0.0084	0.0122	0.0159
	$\bar{L}_t$ 0.9500	0.9986	0.9830	0.9420	0.8914	0.8380	0.7849
	$\bar{T}_t$ 0.0053	0.0013	0	0.0016	0.0057	0.0119	0.0197
15°	$\bar{L}_r$ 0.0356	0.0001	0.0142	0.0464	0.0844	0.1231	0.1604
	$\bar{T}_r$ 0.0086	0.0000	0.0029	0.0096	0.0175	0.0252	0.0320
	$\bar{L}_t$ 0.9438	0.9969	0.9830	0.9403	0.8852	0.8250	0.7631
	$\bar{T}_t$ 0.0120	0.0029	0	0.0037	0.0129	0.0268	0.0446
20°	$\bar{L}_r$ 0.0297	0.0001	0.0122	0.0398	0.0727	0.1069	0.1416
	$\bar{T}_r$ 0.0137	0.0000	0.0048	0.0156	0.0275	0.0382	0.0463
	$\bar{L}_t$ 0.9353	0.9947	0.9830	0.9382	0.8768	0.8071	0.7329
	$\bar{T}_t$ 0.0214	0.0052	0	0.0064	0.0230	0.0478	0.0792
25°	$\bar{L}_r$ 0.0236	0.0000	0.0100	0.0329	0.0615	0.0949	0.1400
	$\bar{T}_r$ 0.0185	0.0000	0.0071	0.0215	0.0360	0.0460	0.0462
	$\bar{L}_t$ 0.9245	0.9917	0.9830	0.9356	0.8666	0.7846	0.6927
	$\bar{T}_t$ 0.0334	0.0082	0	0.0100	0.0359	0.0745	0.1211
30°	$\bar{L}_r$ 0.0181	0.0000	0.0077	0.0267	0.0546	0.1049	0.8853
	$\bar{T}_r$ 0.0222	0.0000	0.0094	0.0262	0.0395	0.0378	0.0452
	$\bar{L}_t$ 0.9118	0.9881	0.9830	0.9327	0.8543	0.7526	0
	$\bar{T}_t$ 0.0480	0.0119	0	0.0143	0.0516	0.1047	0.0695

TABLE 4

AMPLITUDES OF THE TRANSVERSE AND LONGITUDINAL WAVES CREATED AT AN ELASTIC INTERFACE AFTER INCIDENCE OF A TRANSVERSE WAVE

θ		$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
				$\bar{\gamma}=\gamma_2/\gamma_1=0.7$				
0°	S_1/T_1	−0.4815	−0.3115	−0.1765	−0.0667	0.0244	0.1012	0.1667
	R_1/T_1	0	0	0	0	0	0	0
	T_2/T_1	0.7407	0.9836	1.1765	1.3334	1.4634	1.5731	1.6667
	I_2/T_1	0	0	0	0	0	0	0
5°45′14″	S_1/T_1	−0.4561	−0.2953	−0.1692	−0.0681	0.0142	0.0822	0.1388
	R_1/T_1	−0.0329	−0.0210	−0.0116	−0.0039	0.0023	0.0076	0.0120
	T_2/T_1	0.7386	0.9825	1.1764	1.3341	1.4644	1.5735	1.6656
	I_2/T_1	0.0133	0.0093	0.0010	−0.0106	−0.0247	−0.0409	−0.0590
11°23′20″	S_1/T_1	−0.3841	−0.2493	−0.1483	−0.0724	−0.0164	0.0227	0.0454
	R_1/T_1	−0.1229	−0.0785	−0.0441	−0.0172	0.0038	0.0198	0.0309
	T_2/T_1	0.7324	0.9791	1.1764	1.3363	1.4662	1.5711	1.6532
	I_2/T_1	0.0517	0.0365	0.0037	−0.0438	−0.1049	−0.1807	−0.2746
6°46′45″	S_1/T_1	−0.2770	−0.1804	−0.1164	−0.0796	−0.0706	−0.1026	−0.3860
	R_1/T_1	−0.2476	−0.1580	−0.0920	−0.0450	−0.0168	−0.0177	−0.2659
	T_2/T_1	0.7224	0.9736	1.1764	1.3388	1.4630	1.5430	1.5350
	I_2/T_1	0.1108	0.0796	0.0074	−0.1052	−0.2694	−0.5281	−1.4788
				$\bar{\gamma}=\gamma_2/\gamma_1=0.8$				
0°	S_1/T_1	−0.4286	−0.2500	−0.1111	0.0000	0.0909	0.1667	0.2308
	R_1/T_1	0	0	0	0	0	0	0
	T_2/T_1	0.7143	0.9375	1.1111	1.2500	1.3636	1.4584	1.5385
	I_2/T_1	0	0	0	0	0	0	0
5°45′14″	S_1/T_1	−0.4054	−0.2364	−0.1066	−0.0042	0.0782	0.1455	0.2010
	R_1/T_1	−0.0293	−0.0168	−0.0072	0.0004	0.0066	0.0117	0.0160
	T_2/T_1	0.7121	0.9363	1.1111	1.2510	1.3651	1.4597	1.5389
	I_2/T_1	0.0127	0.0086	0.0006	−0.0104	−0.0236	−0.0386	−0.0553
11°23′20″	S_1/T_1	−0.3394	−0.1977	−0.0935	−0.0189	0.0401	0.0792	0.1018
	R_1/T_1	−0.1092	−0.0628	−0.0276	−0.0007	0.0202	0.0358	0.0467
	T_2/T_1	0.7058	0.9328	1.1111	1.2537	1.3686	1.4608	1.5330
	I_2/T_1	0.0494	0.0339	0.0022	−0.0429	−0.1003	−0.1711	−0.2587
16°46′45″	S_1/T_1	−0.2415	−0.1398	−0.0736	−0.0357	−0.0262	−0.0584	−0.3578
	R_1/T_1	−0.2189	−0.1253	−0.0577	−0.0104	0.0176	0.0162	−0.2424
	T_2/T_1	0.6954	0.9269	1.1111	1.2574	1.3695	1.4444	1.4820
	I_2/T_1	0.1060	0.0743	0.0044	−0.1030	−0.2588	−0.5051	−1.4461
				$\bar{\gamma}=\gamma_2/\gamma_1=0.9$				
0°	S_1/T_1	−0.3793	−0.1940	−0.0526	0.0588	0.1489	0.2233	0.2857
	R_1/T_1	0	0	0	0	0	0	0
	T_2/T_1	0.6897	0.8955	1.0526	1.1765	1.2766	1.3592	1.4286
	I_2/T_1	0	0	0	0	0	0	0
5°45′14″	S_1/T_1	−0.3581	−0.1828	−0.0505	0.0523	0.1341	0.2003	0.2545
	R_1/T_1	−0.0258	−0.0131	−0.0034	0.0042	0.0102	0.0152	0.0192
	T_2/T_1	0.6875	0.8943	1.0526	1.1776	1.2784	1.3612	1.4301
	I_2/T_1	0.0121	0.0080	0.0003	−0.0101	−0.0225	−0.0365	−0.0520
11°23′20″	S_1/T_1	−0.2978	−0.1507	−0.0444	0.0334	0.0875	0.1286	0.1509
	R_1/T_1	−0.0965	−0.0486	−0.0130	0.0139	0.0342	0.0497	0.0602
	T_2/T_1	0.6809	0.8906	1.0526	1.1807	1.2832	1.3650	1.4292
	I_2/T_1	0.0473	0.0317	0.0010	−0.0419	−0.0966	−0.1624	−0.2444
16°46′45″	S_1/T_1	−0.2084	−0.1028	−0.0350	0.0035	0.0135	−0.0187	−0.3310
	R_1/T_1	−0.1922	−0.0957	−0.0273	0.0200	0.0476	0.0458	−0.2207
	T_2/T_1	0.6703	0.8845	1.0526	1.1854	1.2872	1.3578	1.4336
	I_2/T_1	0.1016	0.0696	0.0020	−0.1006	−0.2486	−0.4837	−1.4142

$\bar{\gamma}$ = (density of refracting medium)/(density of incident medium) = γ_2/γ_1.
$\bar{\alpha}$ = (velocity in refracting medium)/(velocity in incident medium) = α_2/α_1.
S_1, R_1, T_2, I_2 = amplitudes of wave functions for reflected transverse and longitudinal waves, and transmitted transverse and longitudinal waves; θ = angle of incidence.

TABLE 4—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.0$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	S_1/T_1 −0.3333	−0.1429	0	0.1111	0.2000	0.2727	0.3333
	R_1/T_1 0	0	0	0	0	0	0
	T_2/T_1 0.6667	0.8571	1.0000	1.1111	1.2000	1.2728	1.3333
	I_2/T_1 0	0	0	0	0	0	0
5°45′14″	S_1/T_1 −0.3139	−0.1337	0	0.1026	0.1834	0.2483	0.3011
	R_1/T_1 −0.0229	−0.0098	0	0.0075	0.0135	0.0183	0.0223
	T_2/T_1 0.6645	0.8559	1.0000	1.1123	1.2021	1.2752	1.3356
	I_2/T_1 0.0116	0.0075	0	−0.0099	−0.0215	−0.0346	−0.0490
11°23′20″	S_1/T_1 −0.2590	−0.1077	0	0 .0779	0.1338	0.1721	0.1941
	R_1/T_1 −0.0847	−0.0357	0	0.0267	0.0468	0.0617	0.0719
	T_2/T_1 0.6578	0.8521	1.0000	1.1157	1.2076	1.2810	1.3385
	I_2/T_1 0.0453	0.0297	0	−0.0408	−0.0920	−0.1545	−0.2315
16°46′45″	S_1/T_1 −0.1776−	0.0689	0	0.0389	0.0491	0.0171	−0.3055
	R_1/T_1 −0.1675	−0.0688	0	0.0470	0.0741	0.0719	−0.2005
	T_2/T_1 0.6470	0.8458	1.0000	1.1211	1.2142	1.2811	1.3890
	I_2/T_1 0.0975	0.0655	0	−0.0980	−0.2390	−0.4637	−1.3832
	$\bar{\gamma}=\gamma_2/\gamma_1=1.1$						
0°	S_1/T_1 −0.2903	−0.0959	0.0476	0.1580	0.2453	0.3163	0.3751
	R_1/T_1 0	0	0	0	0	0	0
	T_2/T_1 0.6452	0.8219	0.9524	1.0527	1.1321	1.1967	1.2500
	I_2/T_1 0	0	0	0	0	0	0
5°45′14″	S_1/T_1 −0.2727	−0.0886	0.0457	0.1477	0.2272	0.2906	0.3419
	R_1/T_1 −0.0200	−0.0067	0.0031	0.0105	0.0163	0.0210	0.0248
	T_2/T_1 0.6429	0.8206	0.9524	1.0539	1.1343	1.1994	1.2529
	I_2/T_1 0.0112	0.0070	−0.0002	−0.0098	−0.0206	−0.0329	−0.0464
11°23′20″	S_1/T_1 −0.2232	−0.0681	0.0402	0.1178	0.1731	0.2108	0.2324
	R_1/T_1 −0.0738	−0.0239	0.0117	0.0381	0.0578	0.0722	0.0820
	T_2/T_1 0.6362	0.8168	0.9524	1.0575	1.1406	1.2067	1.2587
	I_2/T_1 0.0435	0.0279	−0.0008	−0.0398	−0.0882	−0.1472	−0.2198
16°46′45″	S_1/T_1 −0.1488	−0.0378	0.0318	0.0709	0.0812	0.0496	−0.2812
	R_1/T_1 −0.1445	−0.0443	0.0246	0.0710	0.0975	0.0951	−0.1816
	T_2/T_1 0.6253	0.8103	0.9524	1.0634	1.1491	1.2126	1.3478
	I_2/T_1 0.0938	0.0619	−0.0016	−0.0955	−0.2300	−0.4451	−1.3531
	$\bar{\gamma}=\gamma_2/\gamma_1=1.2$						
0°	S_1/T_1 −0.2500	−0.0526	0.0909	0.2000	0.2857	0.3549	0.4118
	R_1/T_1 0	0	0	0	0	0	0
	T_2/T_1 0.6250	0.7895	0.9091	1.0000	1.0714	1.1291	1.1765
	I_2/T_1 0	0	0	0	0	0	0
5°45′14″	S_1/T_1 −0.2340	−0.0471	0.0873	0.1882	0.2663	0.3281	0.3780
	R_1/T_1 −0.0173	−0.0039	0.0058	0.0131	0.0188	0.0233	0.0270
	T_2/T_1 0.6227	0.7882	0.9091	1.0013	1.0738	1.1321	1.1798
	I_2/T_1 0.0107	0.0066	−0.0004	−0.0094	−0.0198	−0.0314	−0.0440
11°23′20″	S_1/T_1 −0.1887	−0.0317	0.0769	0.1539	0.2084	0.2454	0.2666
	R_1/T_1 −0.0634	−0.0132	0.0223	0.0483	0.0675	0.0816	0.0910
	T_2/T_1 0.6160	0.7843	0.9091	1.0051	1.0806	1.1405	1.1878
	I_2/T_1 0.0418	0.0263	−0.0015	−0.0387	−0.0848	−0.1406	−0.2092
16°46′45″	S_1/T_1 −0.1219	−0.0090	0.0609	0.1000	0.1104	0.0792	−0.2580
	R_1/T_1 −0.1230	−0.0218	0.0468	0.0926	0.1185	0.1159	−0.1638
	T_2/T_1 0.6050	0.7777	0.9091	1.0114	1.0906	1.1512	1.3095
	I_2/T_1 0.0903	0.0584	−0.0030	−0.0929	−0.2215	−0.4277	−1.3239

TABLE 4—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.3$

θ		$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	S_1/T_1	−0.2121	−0.0127	0.1304	0.2381	0.3220	0.3893	0.4444
	R_1/T_1	0	0	0	0	0	0	0
	T_2/T_1	0.6061	0.7595	0.8696	0.9524	1.0170	1.0688	1.1111
	I_2/T_1	0	0	0	0	0	0	0
5°45′14″	S_1/T_1	−0.1976	−0.0088	0.1253	0.2249	0.3014	0.3617	0.4101
	R_1/T_1	−0.0148	−0.0013	0.0084	0.0155	0.0211	0.0254	0.0290
	T_2/T_1	0.6038	0.7582	0.8696	0.9537	1.0194	1.0720	1.1148
	I_2/T_1	0.0103	0.0063	−0.0005	−0.0091	−0.0190	−0.0300	−0.0419
11°23′20″	S_1/T_1	−0.1567	0.0020	0.1104	0.1866	0.2403	0.2766	0.2973
	R_1/T_1	−0.0538	−0.0033	0.0320	0.0575	0.0762	0.0898	0.0990
	T_2/T_1	0.5970	0.7542	0.8696	0.9576	1.0266	1.0812	1.1245
	I_2/T_1	0.0403	0.0249	−0.0020	−0.0377	−0.0815	−0.1345	−0.1996
16°46′45″	S_1/T_1	−0.0965	0.0176	0.0876	0.1265	0.1370	0.1063	−0.2358
	R_1/T_1	−0.1030	−0.0011	0.0670	0.1121	0.1374	0.1345	−0.1471
	T_2/T_1	0.5859	0.7476	0.8696	0.9642	1.0378	1.0957	1.2736
	I_2/T_1	0.0871	0.0554	−0.0041	−0.0905	−0.2136	−0.4116	−1.2958

TABLE 5

INTENSITIES OF THE TRANSVERSE AND LONGITUDINAL WAVES CREATED AT AN ELASTIC INTERFACE AFTER INCIDENCE OF A TRANSVERSE WAVE

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
			$\bar{\gamma}=\gamma_2/\gamma_1=0.7$				
0°	T_r 0.2318	0.0970	0.0311	0.0044	0.0006	0.0102	0.0278
	L_r 0	0	0	0	0	0	0
	T_t 0.7682	0.9030	0.9689	0.9956	0.9994	0.9898	0.9722
	L_t 0	0	0	0	0	0	0
5°45′14″	T_r 0.2081	0.0872	0.0286	0.0046	0.0002	0.0068	0.0193
	L_r 0.0207	0.0084	0.0025	0.0003	0.0001	0.0011	0.0027
	T_t 0.7638	0.9009	0.9688	0.9967	1.0008	0.9903	0.9709
	L_t 0.0047	0.0015	0.0000	0.0012	0.0054	0.0128	0.0233
11°23′20″	T_r 0.1479	0.0622	0.0220	0.0052	0.0003	0.0005	0.0021
	L_r 0.0746	0.0304	0.0096	0.0015	0.0001	0.0019	0.0047
	T_t 0.7510	0.8947	0.9688	0.9999	1.0032	0.9874	0.9566
	L_t 0.0185	0.0061	0.0000	0.0053	0.0253	0.0645	0.1303
16°46′45″	T_r 0.0767	0.0325	0.0136	0.0063	0.0050	0.0105	0.1490
	L_r 0.1416	0.0577	0.0195	0.0047	0.0007	0.0007	0.1632
	T_t 0.7305	0.8846	0.9687	1.0038	0.9989	0.9524	0.8247
	L_t 0.0397	0.0137	0.0001	0.0143	0.0782	0.2576	1.7675
			$\bar{\gamma}=\gamma_2/\gamma_1=0.8$				
0°	T_r 0.1837	0.0625	0.0123	0.0000	0.0083	0.0278	0.0533
	L_r 0	0	0	0	0	0	0
	T_t 0.8163	0.9375	0.9877	1.0000	0.9917	0.9723	0.9468
	L_t 0	0	0	0	0	0	0
5°45′14″	T_r 0.1643	0.0559	0.0114	0.0000	0.0061	0.0212	0.0404
	L_r 0.0164	0.0054	0.0010	0.0000	0.0008	0.0026	0.0049
	T_t 0.8114	0.9351	0.9876	1.0016	0.9939	0.9740	0.9472
	L_t 0.0049	0.0015	0.0000	0.0013	0.0057	0.0130	0.0234
11°23′20″	T_r 0.1152	0.0391	0.0088	0.0004	0.0016	0.0063	0.0104
	L_r 0.0588	0.0195	0.0038	0.0000	0.0020	0.0063	0.0108
	T_t 0.7970	0.9281	0.9876	1.0059	0.9989	0.9756	0.9401
	L_t 0.0193	0.0061	0.0000	0.0058	0.0265	0.0661	0.1321
16°46′45″	T_r 0.0583	0.0196	0.0054	0.0013	0.0007	0.0034	0.1280
	L_r 0.1106	0.0363	0.0077	0.0002	0.0007	0.0006	0.1357
	T_t 0.7737	0.9164	0.9876	1.0119	1.0002	0.9538	0.8785
	L_t 0.0415	0.0136	0.0000	0.0157	0.0825	0.2694	1.9318
			$\bar{\gamma}=\gamma_2/\gamma_1=0.9$				
0°	T_r 0.1439	0.0376	0.0028	0.0035	0.0222	0.0499	0.0816
	L_r 0	0	0	0	0	0	0
	T_t 0.8561	0.9624	0.9973	0.9966	0.9778	0.9501	0.9184
	L_t 0	0	0	0	0	0	0
5°45′14″	T_r 0.1282	0.0334	0.0026	0.0027	0.0180	0.0401	0.0648
	L_r 0.0128	0.0033	0.0002	0.0003	0.0020	0.0044	0.0071
	T_t 0.8508	0.9597	0.9973	0.9985	0.9806	0.9529	0.9203
	L_t 0.0051	0.0015	0.0000	0.0014	0.0058	0.0131	0.0233
11°23′20″	T_r 0.0887	0.0227	0.0020	0.0011	0.0077	0.0165	0.0228
	L_r 0.0459	0.0117	0.0008	0.0010	0.0058	0.0122	0.0179
	T_t 0.8346	0.9518	0.9973	1.0037	0.9880	0.9582	0.9192
	L_t 0.0198	0.0059	0.0000	0.0062	0.0276	0.0669	0.1327
16°46′45″	T_r 0.0434	0.0106	0.0012	0.0000	0.0002	0.0004	0.1096
	L_r 0.0853	0.0211	0.0017	0.0009	0.0052	0.0048	0.1125
	T_t 0.8087	0.9387	0.9973	1.0117	0.9941	0.9482	0.9248
	L_t 0.0429	0.0134	0.0000	0.0168	0.0856	0.2779	2.0785

$\bar{\gamma}$ = (density of refracting medium)/(density of incident medium) = γ_2/γ_1.
$\bar{\alpha}$ = (velocity in refracting medium)/(velocity in incident medium) = α_2/α_1.
T_r, L_r, T_t, L_t = intensities of reflected transverse, reflected longitudinal, transmitted transverse, and transmitted longitudinal waves per unit intensity of incident transverse waves; θ = angle of incidence.

TABLE 5—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.0$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$		0.75	1.00	1.25	1.50	1.75	2.00
0°	T_r	0.1111	0.0204	0	0.0123	0.0400	0.0744	0.1111
	L_r	0	0	0	0	0	0	0
	T_t	0.8889	0.9795	1.0	0.9877	0.9600	0.9257	0.8889
	L_t	0	0	0	0	0	0	0
5°45′14″	T_r	0.0985	0.0179	0	0.0105	0.0336	0.0617	0.0907
	L_r	0.0100	0.0018	0	0.0011	0.0035	0.0064	0.0095
	T_t	0.8831	0.9768	1.0	0.9898	0.9634	0.9292	0.8919
	L_t	0.0052	0.0014	0	0.0015	0.0059	0.0131	0.0230
11°23′20″	T_r	0.0671	0.0116	0	0.0061	0.0179	0.0296	0.0377
	L_r	0.0354	0.0063	0	0.0035	0.0108	0.0188	0.0255
	T_t	0.8655	0.9681	1.0	0.9958	0.9723	0.9377	0.8958
	L_t	0.0203	0.0058	0	0.0066	0.0278	0.0673	0.1323
16°46′45″	T_r	0.0316	0.0047	0	0.0015	0.0024	0.0003	0.0933
	L_r	0.0648	0.0109	0	0.0051	0.0127	0.0119	0.0928
	T_t	0.8373	0.9538	1.0	1.0055	0.9829	0.9378	0.9647
	L_t	0.0439	0.0132	0	0.0177	0.0879	0.2838	2.2092
				$\bar{\gamma}=\gamma_2/\gamma_1=1.1$				
0°	T_r	0.0843	0.0092	0.0023	0.0250	0.0602	0.1000	0.1407
	L_r	0	0	0	0	0	0	0
	T_t	0.9158	0.9908	0.9978	0.9751	0.9399	0.9001	0.8594
	L_t	0	0	0	0	0	0	0
5°45′14″	T_r	0.0744	0.0079	0.0021	0.0218	0.0516	0.0844	0.1169
	L_r	0.0077	0.0009	0.0002	0.0021	0.0051	0.0084	0.0119
	T_t	0.9093	0.9876	0.9978	0.9774	0.9435	0.9042	0.8634
	L_t	0.0053	0.0014	0.0000	0.0016	0.0060	0.0130	0.0226
11°23′20″	T_r	0.0498	0.0046	0.0016	0.0139	0.0300	0.0444	0.0540
	L_r	0.0269	0.0028	0.0007	0.0072	0.0165	0.0257	0.0332
	T_t	0.8905	0.9785	0.9978	0.9841	0.9540	0.9153	0.8714
	L_t	0.0205	0.0056	0.0000	0.0069	0.0282	0.0673	0.1311
16°46′45″	T_r	0.0221	0.0014	0.0010	0.0050	0.0066	0.0025	0.0791
	L_r	0.0482	0.0045	0.0014	0.0116	0.0220	0.0209	0.0762
	T_t	0.8602	0.9630	0.9978	0.9951	0.9683	0.9243	0.9991
	L_t	0.0447	0.0130	0.0000	0.0185	0.0896	0.2876	2.3255
				$\bar{\gamma}=\gamma_2/\gamma_1=1.2$				
0°	T_r	0.0625	0.0028	0.0083	0.0400	0.0816	0.1260	0.1696
	L_r	0	0	0	0	0	0	0
	T_t	0.9375	0.9973	0.9918	0.9600	0.9184	0.8742	0.8305
	L_t	0	0	0	0	0	0	0
5°45′14″	T_r	0.0548	0.0022	0.0076	0.0354	0.0709	0.1077	0.1429
	L_r	0.0057	0.0003	0.0007	0.0033	0.0068	0.0104	0.0140
	T_t	0.9307	0.9940	0.9918	0.9625	0.9224	0.8788	0.8352
	L_t	0.0052	0.0013	0.0000	0.0016	0.0060	0.0129	0.0223
11°23′20″	T_r	0.0356	0.0010	0.0059	0.0237	0.0434	0.0602	0.0711
	L_r	0.0198	0.0009	0.0025	0.0115	0.0225	0.0329	0.0409
	T_t	0.9107	0.9842	0.9918	0.9698	0.9342	0.8919	0.8465
	L_t	0.0207	0.0055	0.0000	0.0071	0.0284	0.0669	0.1296
16°46′45″	T_r	0.0149	0.0001	0.0037	0.0100	0.0122	0.0063	0.0666
	L_r	0.0350	0.0011	0.0051	0.0198	0.0324	0.0310	0.0620
	T_t	0.8785	0.9677	0.9918	0.9820	0.9515	0.9087	1.0289
	L_t	0.0452	0.0126	0.0000	0.0191	0.0906	0.2897	2.4286

TABLE 5—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.3$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$		0.75	1.00	1.25	1.50	1.75	2.00
0°	T_r	0.0450	0.0002	0.0170	0.0567	0.1037	0.1515	0.1975
	L_r	0	0	0	0	0	0	0
	T_t	0.9551	0.9999	0.9830	0.9433	0.8964	0.8486	0.8025
	L_t	0	0	0	0	0	0	0
5°45′14″	T_r	0.0391	0.0001	0.0157	0.0506	0.0909	0.1308	0.1682
	L_r	0.0042	0.0000	0.0014	0.0046	0.0085	0.0124	0.0161
	T_t	0.9479	0.9964	0.9830	0.9459	0.9006	0.8537	0.8078
	L_t	0.0053	0.0013	0.0000	0.0017	0.0060	0.0128	0.0219
11°23′20″	T_r	0.0246	0.0000	0.0122	0.0348	0.0577	0.0765	0.0884
	L_r	0.0143	0.0001	0.0051	0.0163	0.0287	0.0398	0.0484
	T_t	0.9267	0.9859	0.9830	0.9537	0.9134	0.8684	0.8219
	L_t	0.0208	0.0053	0.0000	0.0073	0.0284	0.0663	0.1278
16°46′45″	T_r	0.0093	0.0003	0.0077	0.0160	0.0188	0.0113	0.0556
	L_r	0.0245	0.0000	0.0104	0.0290	0.0436	0.0418	0.0500
	T_t	0.8925	0.9688	0.9830	0.9669	0.9334	0.8918	1.0543
	L_t	0.0456	0.0123	0.0000	0.0197	0.0913	0.2906	2.5205

TABLE 6

FRACTIONS OF THE INCIDENT ENERGY TRANSFORMED INTO THE TRANSVERSE AND LONGITUDINAL WAVES CREATED AT AN ELASTIC INTERFACE AFTER INCIDENCE OF A TRANSVERSE WAVE

θ	$\bar{\alpha}=\alpha_2/\alpha_1=$	0.50	0.75	1.00	1.25	1.50	1.75	2.00
				$\bar{\gamma}=\gamma_2/\gamma_1=0.7$				
0°	$\bar{T}_r$	0.2318	0.0970	0.0311	0.0044	0.0006	0.0102	0.0278
	$\bar{L}_r$	0	0	0	0	0	0	0
	$\bar{T}_t$	0.7682	0.9030	0.9689	0.9956	0.9994	0.9898	0.9722
	$\bar{L}_t$	0	0	0	0	0	0	0
5°45′14″	$\bar{T}_r$	0.2081	0.0872	0.0286	0.0046	0.0002	0.0068	0.0193
	$\bar{L}_r$	0.0205	0.0083	0.0025	0.0003	0.0001	0.0011	0.0027
	$\bar{T}_t$	0.7667	0.9029	0.9688	0.9939	0.9945	0.9799	0.9560
	$\bar{L}_t$	0.0047	0.0015	0.0000	0.0012	0.0052	0.0123	0.0220
11°23′20″	$\bar{T}_r$	0.1476	0.0622	0.0220	0.0052	0.0003	0.0005	0.0021
	$\bar{L}_r$	0.0715	0.0292	0.0092	0.0014	0.0001	0.0018	0.0045
	$\bar{T}_t$	0.7623	0.9026	0.9688	0.9884	0.9774	0.9450	0.8964
	$\bar{L}_t$	0.0186	0.0060	0.0000	0.0049	0.0222	0.0527	0.0970
16°46′45″	$\bar{T}_r$	0.0767	0.0325	0.0136	0.0063	0.0050	0.0105	0.1490
	$\bar{L}_r$	0.1281	0.0522	0.0176	0.0042	0.0006	0.0007	0.1476
	$\bar{T}_t$	0.7550	0.9021	0.9687	0.9778	0.9404	0.8585	0.7033
	$\bar{L}_t$	0.0402	0.0132	0.0001	0.0117	0.0540	0.1302	0
				$\bar{\gamma}=\gamma_2/\gamma_1=0.8$				
0°	$\bar{T}_r$	0.1837	0.0625	0.0123	0.0000	0.0083	0.0278	0.0533
	$\bar{L}_r$	0	0	0	0	0	0	0
	$\bar{T}_t$	0.8163	0.9375	0.9877	1.0000	0.9917	0.9723	0.9468
	$\bar{L}_t$	0	0	0	0	0	0	0
5°45′14″	$\bar{T}_r$	0.1643	0.0559	0.0114	0.0000	0.0061	0.0212	0.0404
	$\bar{L}_r$	0.0163	0.0054	0.0010	0.0000	0.0008	0.0026	0.0049
	$\bar{T}_t$	0.8145	0.9372	0.9876	0.9987	0.9876	0.9637	0.9326
	$\bar{L}_t$	0.0049	0.0015	0.0000	0.0013	0.0056	0.0125	0.0221
11°23′20″	$\bar{T}_r$	0.1152	0.0391	0.0088	0.0004	0.0016	0.0063	0.0104
	$\bar{L}_r$	0.0564	0.0187	0.0036	0.0000	0.0019	0.0061	0.0104
	$\bar{T}_t$	0.8090	0.9363	0.9876	0.9943	0.9733	0.9337	0.8810
	$\bar{L}_t$	0.0194	0.0060	0.0000	0.0054	0.0232	0.0540	0.0983
16°46′45″	$\bar{T}_r$	0.0583	0.0196	0.0054	0.0013	0.0007	0.0034	0.1280
	$\bar{L}_r$	0.1001	0.0328	0.0070	0.0002	0.0006	0.0005	0.1228
	$\bar{T}_t$	0.7996	0.9344	0.9876	0.9857	0.9417	0.8598	0.7492
	$\bar{L}_t$	0.0420	0.0132	0.0000	0.0128	0.0570	0.1362	0
				$\bar{\gamma}=\gamma_2/\gamma_1=0.9$				
0°	$\bar{T}_r$	0.1439	0.0376	0.0028	0.0035	0.0222	0.0499	0.0816
	$\bar{L}_r$	0	0	0	0	0	0	0
	$\bar{T}_t$	0.8561	0.9624	0.9973	0.9966	0.9778	0.9501	0.9184
	$\bar{L}_t$	0	0	0	0	0	0	0
5°45′14″	$\bar{T}_r$	0.1282	0.0334	0.0026	0.0027	0.0180	0.0401	0.0648
	$\bar{L}_r$	0.0127	0.0033	0.0002	0.0003	0.0020	0.0044	0.0070
	$\bar{T}_t$	0.8540	0.9618	0.9973	0.9956	0.9744	0.9429	0.9062
	$\bar{L}_t$	0.0051	0.0015	0.0000	0.0014	0.0056	0.0125	0.0220
11°23′20″	$\bar{T}_r$	0.0887	0.0227	0.0020	0.0011	0.0077	0.0165	0.0228
	$\bar{L}_r$	0.0441	0.0112	0.0008	0.0010	0.0056	0.0117	0.0172
	$\bar{T}_t$	0.8472	0.9602	0.9973	0.9922	0.9626	0.9171	0.8614
	$\bar{L}_t$	0.0200	0.0059	0.0000	0.0057	0.0242	0.0547	0.0987
16°46′45″	$\bar{T}_r$	0.0434	0.0106	0.0012	0.0000	0.0002	0.0004	0.1096
	$\bar{L}_r$	0.0772	0.0191	0.0015	0.0008	0.0047	0.0044	0.1018
	$\bar{T}_t$	0.8359	0.9572	0.9973	0.9855	0.9359	0.8547	0.7887
	$\bar{L}_t$	0.0434	0.0130	0.0000	0.0137	0.0592	0.1405	0

$\bar{\gamma}$ (density of refracting medium)/(density of incident medium) $=\gamma_2/\gamma_1$.
$\bar{\alpha}$ = (velocity in refracting medium)/(velocity in incident medium) $=\alpha_2/\alpha_1$.
$\bar{T}_r$, $\bar{L}_r$, $\bar{T}_t$, $\bar{L}_t$ = fractions of incident energy transformed into reflected transverse, reflected longitudinal, transmitted transverse, and transmitted longitudinal waves; θ = angle of incidence.

TABLE 6—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.0$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$		0.75	1.00	1.25	1.50	1.75	2.00
0°	$\bar{T}_r$	0.1111	0.0204	0	0.0123	0.0400	0.0744	0.1111
	$\bar{L}_r$	0	0	0	0	0	0	0
	$\bar{T}_t$	0.8889	0.9795	1.0	0.9877	0.9600	0.9257	0.8889
	$\bar{L}_t$	0	0	0	0	0	0	0
5°45′14″	$\bar{T}_r$	0.0985	0.0179	0	0.0105	0.0336	0.0617	0.0907
	$\bar{L}_r$	0.0099	0.0018	0	0.0011	0.0035	0.0063	0.0094
	$\bar{T}_t$	0.8865	0.9789	1.0	0.9870	0.9573	0.9194	0.8782
	$\bar{L}_t$	0.0052	0.0014	0	0.0015	0.0057	0.0125	0.0217
11°23′20″	$\bar{T}_r$	0.0671	0.0116	0	0.0061	0.0179	0.0296	0.0377
	$\bar{L}_r$	0.0339	0.0060	0	0.0034	0.0104	0.0180	0.0244
	$\bar{T}_t$	0.8786	0.9767	1.0	0.9844	0.9473	0.8974	0.8395
	$\bar{L}_t$	0.0204	0.0057	0	0.0061	0.0244	0.0550	0.0984
16°46′45″	$\bar{T}_r$	0.0316	0.0047	0	0.0015	0.0024	0.0003	0.0933
	$\bar{L}_r$	0.0586	0.0099	0	0.0046	0.0115	0.0108	0.0839
	$\bar{T}_t$	0.8654	0.9726	1.0	0.9795	0.9253	0.8454	0.8227
	$\bar{L}_t$	0.0444	0.0128	0	0.0144	0.0607	0.1435	0
				$\bar{\gamma}=\gamma_2/\gamma_1=1.1$				
0°	$\bar{T}_r$	0.0843	0.0092	0.0023	0.0250	0.0602	0.1000	0.1407
	$\bar{L}_r$	0	0	0	0	0	0	0
	$\bar{T}_t$	0.9158	0.9908	0.9978	0.9751	0.9399	0.9001	0.8594
	$\bar{L}_t$	0	0	0	0	0	0	0
5°45′14″	$\bar{T}_r$	0.0744	0.0079	0.0021	0.0218	0.0516	0.0844	0.1169
	$\bar{L}_r$	0.0076	0.0009	0.0002	0.0021	0.0051	0.0083	0.0117
	$\bar{T}_t$	0.9127	0.9898	0.9978	0.9746	0.9375	0.8947	0.8501
	$\bar{L}_t$	0.0053	0.0014	0.0000	0.0016	0.0058	0.0125	0.0213
11°23′20″	$\bar{T}_r$	0.0498	0.0046	0.0016	0.0139	0.0300	0.0444	0.0540
	$\bar{L}_r$	0.0258	0.0027	0.0007	0.0069	0.0158	0.0246	0.0318
	$\bar{T}_t$	0.9039	0.9872	0.9978	0.9728	0.9295	0.8760	0.8166
	$\bar{L}_t$	0.0206	0.0055	0.0000	0.0064	0.0247	0.0550	0.0976
16°46′45″	$\bar{T}_r$	0.0221	0.0014	0.0010	0.0050	0.0066	0.0025	0.0791
	$\bar{L}_r$	0.0436	0.0041	0.0013	0.0105	0.0199	0.0189	0.0689
	$\bar{T}_t$	0.8891	0.9820	0.9978	0.9693	0.9116	0.8332	0.8520
	$\bar{L}_t$	0.0452	0.0126	0.0000	0.0151	0.0619	0.1454	0
				$\bar{\gamma}=\gamma_2/\gamma_1=1.2$				
0°	$\bar{T}_r$	0.0625	0.0028	0.0083	0.0400	0.0816	0.1259	0.1696
	$\bar{L}_r$	0	0	0	0	0	0	0
	$\bar{T}_t$	0.9375	0.9973	0.9918	0.9600	0.9184	0.8742	0.8305
	$\bar{L}_t$	0	0	0	0	0	0	0
5°45′14″	$\bar{T}_r$	0.0548	0.0022	0.0076	0.0354	0.0709	0.1077	0.1429
	$\bar{L}_r$	0.0057	0.0003	0.0007	0.0033	0.0067	0.0103	0.0139
	$\bar{T}_t$	0.9342	0.9962	0.9918	0.9597	0.9166	0.8696	0.8223
	$\bar{L}_t$	0.0052	0.0013	0.0000	0.0016	0.0058	0.0124	0.0210
11°23′20″	$\bar{T}_r$	0.0356	0.0010	0.0059	0.0237	0.0434	0.0602	0.0711
	$\bar{L}_r$	0.0190	0.0008	0.0024	0.0110	0.0216	0.0315	0.0392
	$\bar{T}_t$	0.9245	0.9929	0.9918	0.9587	0.9102	0.8536	0.7933
	$\bar{L}_t$	0.0208	0.0054	0.0000	0.0065	0.0249	0.0547	0.0964
16°46′45″	$\bar{T}_r$	0.0149	0.0001	0.0037	0.0100	0.0122	0.0063	0.0666
	$\bar{L}_r$	0.0316	0.0010	0.0046	0.0179	0.0293	0.0280	0.0561
	$\bar{T}_t$	0.9078	0.9868	0.9918	0.9566	0.8958	0.8191	0.8774
	$\bar{L}_t$	0.0457	0.0122	0.0000	0.0156	0.0626	0.1465	0

TABLE 6—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.3$

θ	$\bar{\alpha}=\alpha_2/\alpha_1=0.50$		0.75	1.00	1.25	1.50	1.75	2.00
0°	$\overline{T}_r$	0.0450	0.0001	0.0170	0.0567	0.1037	0.1515	0.1975
	$\overline{L}_r$	0	0	0	0	0	0	0
	$\overline{T}_t$	0.9551	0.9999	0.9830	0.9433	0.8964	0.8486	0.8025
	$\overline{L}_t$	0	0	0	0	0	0	0
5°45′14″	$\overline{T}_r$	0.0391	0.0001	0.0157	0.0506	0.0909	0.1308	0.1682
	$\overline{L}_r$	0.0042	0.0000	0.0014	0.0046	0.0084	0.0123	0.0159
	$\overline{T}_t$	0.9515	0.9986	0.9830	0.9432	0.8949	0.8447	0.7954
	$\overline{L}_t$	0.0053	0.0013	0.0000	0.0017	0.0058	0.0123	0.0206
11°23′20″	$\overline{T}_r$	0.0246	0.0000	0.0122	0.0348	0.0577	0.0765	0.0884
	$\overline{L}_r$	0.0137	0.0001	0.0049	0.0156	0.0275	0.0382	0.0464
	$\overline{T}_t$	0.9407	0.9947	0.9830	0.9428	0.8899	0.8311	0.7702
	$\overline{L}_t$	0.0209	0.0052	0.0000	0.0067	0.0249	0.0542	0.0951
16°46′45″	$\overline{T}_r$	0.0093	0.0003	0.0077	0.0160	0.0188	0.0113	0.0556
	$\overline{L}_r$	0.0222	0.0000	0.0094	0.0262	0.0394	0.0378	0.0452
	$\overline{T}_t$	0.9224	0.9879	0.9830	0.9418	0.8787	0.8039	0.8991
	$\overline{L}_t$	0.0461	0.0119	0.0000	0.0161	0.0631	0.1469	0

TABLE 7

REFLECTION COEFFICIENTS, S_1^2/T_1^2, FOR THE INCIDENCE UPON AN ELASTIC INTERFACE OF TRANSVERSE WAVES POLARIZED NORMAL TO THE PLANE OF INCIDENCE

θ	$\bar{\beta}=\beta_2/\beta_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
			$\bar{\gamma}=\gamma_2/\gamma_1=0.7$				
0°	0.2318	0.0970	0.0311	0.0044	0.0006	0.0102	0.0278
5°	0.2308	0.0965	0.0311	0.0047	0.0005	0.0094	0.0259
10°	0.2276	0.0951	0.0311	0.0050	0.0002	0.0072	0.0204
15°	0.2222	0.0923	0.0311	0.0059	0.0000	0.0038	0.0115
20°	0.2144	0.0892	0.0311	0.0074	0.0004	0.0005	0.0017
25°	0.2043	0.0846	0.0311	0.0098	0.0028	0.0022	0.0091
30°	0.1914	0.0786	0.0311	0.0140	0.0113	0.0350	1.0000
34°51′						1.0000	
35°	0.1757	0.0713	0.0311	0.0214	0.0409		
40°	0.1568	0.0623	0.0311	0.0363	0.1932		
41°48′					1.0000		
45°	0.1347	0.0517	0.0311	0.0711			
50°	0.1093	0.0395	0.0311	0.1904			
53°7′				1.0000			
55°	0.0811	0.0260	0.0311				
60°	0.0512	0.0126	0.0311				
65°	0.0227	0.0022	0.0311				
70°	0.0026	0.0018	0.0311				
75°	0.0071	0.0276	0.0311				
80°	0.0750	0.1168	0.0311				
85°	0.3067	0.3604	0.0311				
90°	1.0000	1.0000	0.0311				
			$\bar{\gamma}=\gamma_2/\gamma_1=0.8$				
0°	0.1837	0.0625	0.0123	0.0000	0.0083	0.0278	0.0533
5°	0.1827	0.0621	0.0123	0.0000	0.0078	0.0265	0.0507
10°	0.1796	0.0609	0.0123	0.0000	0.0066	0.0226	0.0430
15°	0.1746	0.0586	0.0123	0.0001	0.0046	0.0163	0.0298
20°	0.1673	0.0561	0.0123	0.0004	0.0022	0.0078	0.0117
25°	0.1578	0.0522	0.0123	0.0011	0.0002	0.0004	0.0008
30°	0.1459	0.0474	0.0123	0.0027	0.0016	0.0149	1.0000
34°51′						1.0000	
35°	0.1315	0.0416	0.0123	0.0065	0.0189		
40°	0.1144	0.0346	0.0123	0.0157	0.1475		
41°48′					1.0000		
45°	0.0948	0.0266	0.0123	0.0415			
50°	0.0729	0.0179	0.0123	0.1450			
53°7′				1.0000			
55°	0.0494	0.0092	0.0123				
60°	0.0263	0.0021	0.0123				
65°	0.0072	0.0004	0.0123				
70°	0.0003	0.0119	0.0123				
75°	0.0225	0.0530	0.0123				
80°	0.1118	0.1594	0.0123				
85°	0.3580	0.4113	0.0123				
90°	1.0000	1.0000	0.0123				

$\bar{\gamma}$ = (density of refracting medium)/(density of incident medium) = γ_2/γ_1.
$\bar{\beta}$ = (velocity in refracting medium)/(velocity in incident medium) = β_2/β_1.
S_1^2/T_1^2 = fractions of the incident energy which is reflected at the interface.
θ = angle of incidence.

TABLE 7—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=0.9$

θ	$\bar{\beta}=\beta_2/\beta_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	0.1439	0.0376	0.0028	0.0035	0.0222	0.0499	0.0816
5°	0.1429	0.0373	0.0028	0.0032	0.0215	0.0482	0.0786
10°	0.1401	0.0364	0.0028	0.0030	0.0194	0.0431	0.0692
15°	0.1354	0.0346	0.0028	0.0024	0.0160	0.0342	0.0525
20°	0.1288	0.0326	0.0028	0.0016	0.0111	0.0214	0.0275
25°	0.1201	0.0296	0.0028	0.0007	0.0053	0.0061	0.0009
30°	0.1093	0.0259	0.0028	0.0000	0.0004	0.0040	1.0000
34°51′						1.0000	
35°	0.0963	0.0216	0.0028	0.0005	0.0063		
40°	0.0813	0.0166	0.0028	0.0045	0.1107		
41°48′					1.0000		
45°	0.0644	0.0111	0.0028	0.0215			
50°	0.0460	0.0057	0.0028	0.1085			
53°7′				1.0000			
55°	0.0274	0.0014	0.0028				
60°	0.0109	0.0002	0.0028				
65°	0.0007	0.0062	0.0028				
70°	0.0056	0.0278	0.0028				
75°	0.0429	0.0813	0.0028				
80°	0.1487	0.2003	0.0028				
85°	0.4030	0.4552	0.0028				
90°	1.0000	1.0000	0.0028				

$\bar{\gamma}=\gamma_2/\gamma_1=1.0$

0°	0.1111	0.0204	0	0.0123	0.0400	0.0744	0.1111
5°	0.1103	0.0202	0	0.0119	0.0391	0.0724	0.1077
10°	0.1077	0.0195	0	0.0114	0.0364	0.0662	0.0970
15°	0.1034	0.0181	0	0.0102	0.0316	0.0553	0.0775
20°	0.0974	0.0167	0	0.0085	0.0247	0.0390	0.0469
25°	0.0896	0.0145	0	0.0062	0.0155	0.0170	0.0068
30°	0.0800	0.0120	0	0.0036	0.0051	0.0001	1.0000
34°51′						1.0000	
35°	0.0687	0.0090	0	0.0010	0.0007		
40°	0.0557	0.0059	0	0.0002	0.0813		
41°48′					1.0000		
45°	0.0415	0.0028	0	0.0090			
50°	0.0268	0.0005	0	0.0793			
53°7′				1.0000			
55°	0.0130	0.0002	0				
60°	0.0027	0.0043	0				
65°	0.0007	0.0172	0				
70°	0.0161	0.0473	0				
75°	0.0660	0.1107	0				
80°	0.1845	0.2388	0				
85°	0.4426	0.4933	0				
90°	1.0000	1.0000	0				

TABLE 7—*Continued* $\bar{\gamma}=\gamma_2/\gamma_1=1.1$

θ	$\bar{\beta}=\beta_2/\beta_1=0.50$	0.75	1.00	1.25	1.50	1.75	2.00
0°	0.0843	0.0092	0.0023	0.0249	0.0602	0.1000	0.1406
5°	0.0835	0.0090	0.0023	0.0243	0.0591	0.0978	0.1369
10°	0.0812	0.0086	0.0023	0.0236	0.0558	0.0907	0.1252
15°	0.0774	0.0077	0.0023	0.0219	0.0500	0.0783	0.1035
20°	0.0721	0.0067	0.0023	0.0193	0.0413	0.0589	0.0683
25°	0.0652	0.0054	0.0023	0.0159	0.0293	0.0313	0.0167
30°	0.0569	0.0039	0.0023	0.0114	0.0141	0.0013	1.0000
34°51′						1.0000	
35°	0.0471	0.0023	0.0023	0.0061	0.0004		
40°	0.0363	0.0008	0.0023	0.0011	0.0580		
41°48′					1.0000		
45°	0.0249	0.0000	0.0023	0.0022			
50°	0.0137	0.0006	0.0023	0.0562			
53°7′				1.0000			
55°	0.0045	0.0040	0.0023				
60°	0.0000	0.0128	0.0023				
65°	0.0055	0.0315	0.0023				
70°	0.0301	0.0688	0.0023				
75°	0.0905	0.1402	0.0023				
80°	0.2187	0.2747	0.0023				
85°	0.4775	0.5266	0.0023				
90°	1.0000	1.0000	0.0023				
			$\bar{\gamma}=\gamma_2/\gamma_1=1.2$				
0°	0.0625	0.0028	0.0083	0.0400	0.0816	0.1259	0.1695
5°	0.0618	0.0027	0.0083	0.0392	0.0804	0.1234	0.1656
10°	0.0598	0.0024	0.0083	0.0383	0.0766	0.1158	0.1531
15°	0.0565	0.0020	0.0083	0.0361	0.0699	0.1020	0.1297
20°	0.0518	0.0015	0.0083	0.0329	0.0598	0.0802	0.0909
25°	0.0459	0.0009	0.0083	0.0284	0.0454	0.0478	0.0295
30°	0.0388	0.0003	0.0083	0.0224	0.0260	0.0064	1.0000
34°51′						1.0000	
35°	0.0307	0.0000	0.0083	0.0148	0.0041		
40°	0.0220	0.0002	0.0083	0.0059	0.0398		
41°48′					1.0000		
45°	0.0132	0.0014	0.0083	0.0000			
50°	0.0055	0.0046	0.0083	0.0383			
53°7′				1.0000			
55°	0.0005	0.0113	0.0083				
60°	0.0015	0.0242	0.0083				
65°	0.0137	0.0481	0.0083				
70°	0.0463	0.0914	0.0083				
75°	0.1155	0.1691	0.0083				
80°	0.2509	0.3080	0.0083				
85°	0.5084	0.5560	0.0083				
90°	1.0000	1.0000	0.0083				
			$\bar{\gamma}=\gamma_2/\gamma_1=1.3$				
0°	0.0450	0.0002	0.0170	0.0567	0.1037	0.1516	0.1975
5°	0.0444	0.0001	0.0170	0.0557	0.1023	0.1489	0.1934
10°	0.0427	0.0001	0.0170	0.0547	0.0982	0.1407	0.1803
15°	0.0398	0.0000	0.0170	0.0521	0.0908	0.1259	0.1556
20°	0.0359	0.0000	0.0170	0.0483	0.0794	0.1021	0.1138
25°	0.0309	0.0001	0.0170	0.0429	0.0630	0.0657	0.0443
30°	0.0250	0.0005	0.0170	0.0356	0.0400	0.0143	1.0000
34°51′						1.0000	
35°	0.0186	0.0013	0.0170	0.0258	0.0108		
40°	0.0119	0.0030	0.0170	0.0135	0.0258		
41°48′					1.0000		
45°	0.0057	0.0061	0.0170	0.0013			
50°	0.0012	0.0116	0.0170	0.0246			
53°7′				1.0000			
55°	0.0003	0.0212	0.0170				
60°	0.0063	0.0378	0.0170				
65°	0.0245	0.0661	0.0170				
70°	0.0640	0.1144	0.0170				
75°	0.1405	0.1971	0.0170				
80°	0.2812	0.3388	0.0170				
85°	0.5360	0.5820	0.0170				
90°	1.0000	1.0000	0.0170				

THE SEISMIC WAVE ENERGY REFLECTED FROM VARIOUS TYPES OF STRATIFIED HORIZONS*

M. MUSKAT† AND M. W. MERES†

ABSTRACT

Two applications are made of the reflection and transmission coefficients reported in the preceding paper. These concern the effect of the angle of incidence upon the fraction of incident energy returning to the surface, and the effect of velocity stratification upon the energy return.

In the present stage of development of the art of seismic prospecting it is not feasible to base the interpretation methods upon the amplitudes or intensities of the waves recorded at the surface. Among the reasons for this situation are the following: (1) the intensity distribution among the waves of various frequencies in the explosion or source of the seismic waves is not known; (2) the geometrical attenuation due to the spreading of the waves is rather difficult to take into account because of the inhomogeneity of the media being traversed; (3) the direct absorption properties of the sedimentary strata are not known quantitatively; and (4) the effect of the detecting and recording mechanism upon the amplitudes of the waves as they reach the surface would involve extended experimentation for their determination.

The final wave amplitudes as they appear on a seismograph record are thus the resultant of a multitude of extraneous effects superposed upon the direct reflection and refraction processes occurring at the horizons being studied. In fact these extraneous factors may predominate in determining the final amplitude record on the seismogram, and they may mask completely the significant features which are characteristic of the phenomena of direct geophysical interest. It is, therefore, impossible to infer from the fact that the amplitude of a certain wave may be large or small that the reflection or refraction process of interest has a corresponding behavior.

A knowledge of the reflection and refraction coefficients for various interfaces is nevertheless of interest in providing a background for qualitative considerations regarding the possibilities of obtaining satisfactory records under specified conditions. Thus if the interface at the horizon of interest should be such that the reflection coefficient

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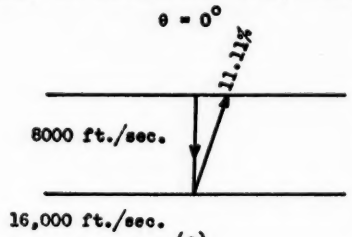
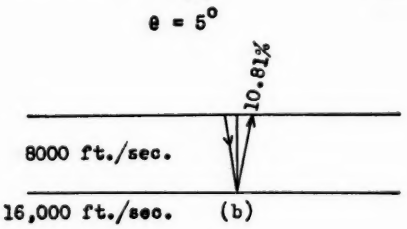
at the interface should be extremely small, one could conclude that it would be difficult to follow that horizon by the ordinary reflection shooting processes even though practically nothing is known about the other extraneous factors which would also enter the problem. A few typical examples have been carried through from this point of view to show some of the more simple implications of the body of reflection and transmission coefficients previously computed.[1]

The specific examples have been chosen so as to provide an answer to two specific questions concerning reflection processes: (1) what is the effect of the angle of incidence, and (2) what is the effect of multiple stratification? Both the nature of the specific cases and the results may be best presented by means of diagrams. They are as follows:[2]



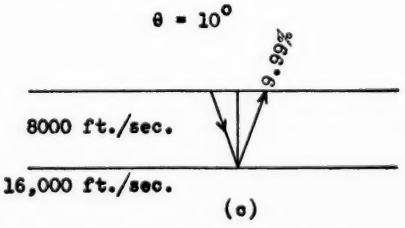
(a) Normal incidence on an interface separating an incident bed with velocity 8000 ft./sec. from a refracting bed of velocity 16,000 ft./sec.

11.1% of the incident energy will be reflected to the surface from the interface as a longitudinal wave.



(b) Same as case (a) but with a 5° angle of incidence.

10.8% of the incident energy will be reflected.



(c) Same as case (a) but with an angle of incidence equal to 10°.

10.0% of the incident energy will be reflected.

[1] See the preceding paper by M. Muskat and M. W. Meres.

[2] In all these cases it has been assumed that the incident wave is longitudinal. Moreover, only the energy of the various reflected longitudinal waves were computed explicitly, although the transformation of the longitudinal wave energy to transverse wave energy has been taken into account in the original calculation of the reflection and transmission coefficients. The densities of all the various strata have been taken as equal.

(d) A two interface system in which the incident stratum of velocity 8000 ft./sec. is separated from the final reflecting stratum of velocity 16,000 ft./sec. by a bed of intermediate velocity, 12,000 ft./sec. The angle of incidence = 0°.

Energy reflected from first interface equals 4% of incident energy; energy returning from the highest speed layer = 2.3% of incident energy.

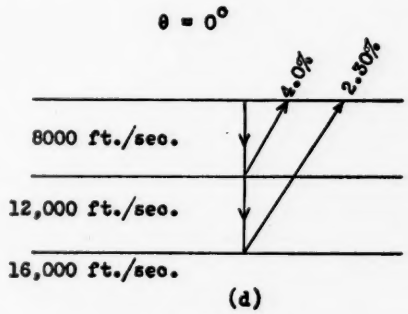
(e) Same as case (d) but with angle of incidence at the second interface = 5°.

Energy reflected from first interface = 3.9%; energy returning from highest speed layer = 1.8% of incident energy.

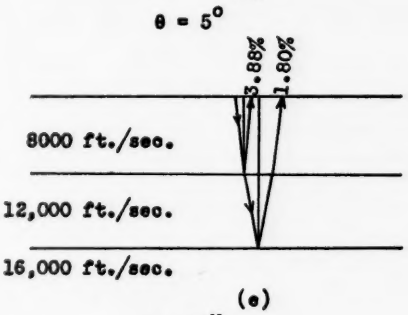
(f) Same as case (d) but with angle of incidence = 10°.

Energy reflected from first interface = 3.8%; energy returning from highest speed layer = 1.7%.

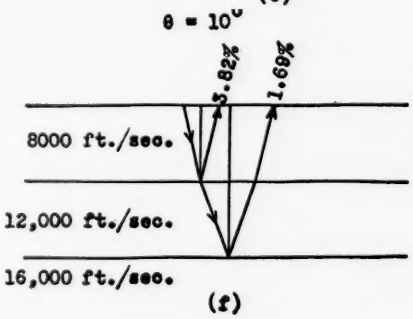
(g) Same as case (d) but with angles of incidence so adjusted that the effective angle of incidence, θ_e for a single layer would be 10°. These angles were determined by the formulas: $2 \tan \theta_e = \tan \theta_i + \tan \theta_r$; $\sin \theta_i / \sin \theta_r = 2/3$, the two beds being taken of equal thickness, and θ_i, θ_r repre-



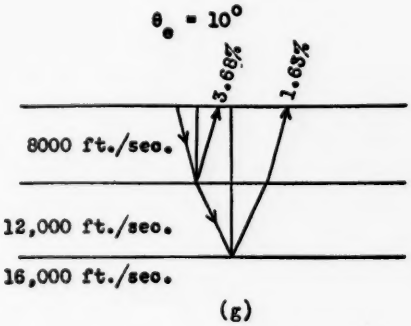
(d)



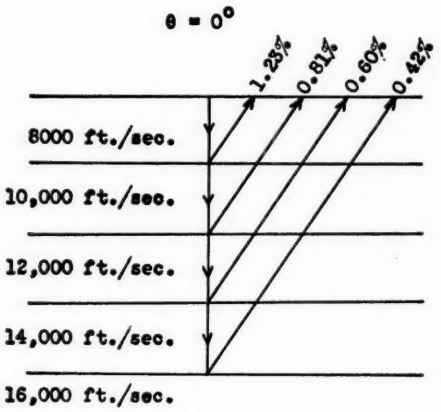
(e)



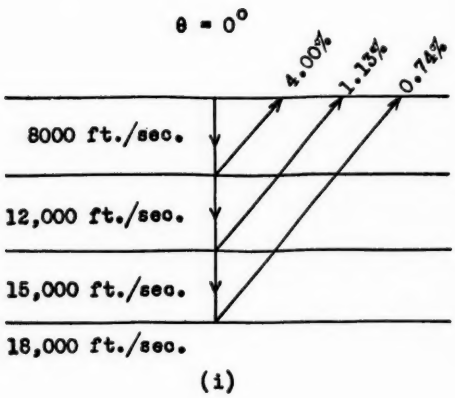
(f)



(g)



(h)



(i)

senting the angles of incidence and refraction at the first interface.

Energy reflected from first interface = 3.7%; fraction of incident energy returning from highest speed layer = 1.6%.

(h) A 4-interface system in which the highest and lowest velocity beds are separated by three intermediate layers with equal velocity increments. The angle of incidence = 0°.

Energy reflected from 10,000 ft. bed = 1.2% of incident energy; energy reflected from 12,000 ft. bed = 0.8% of incident energy; energy reflected from 14,000 ft. bed = 0.6% of incident energy; energy reflected from highest speed bed = 0.4% of incident energy.

(i) Reflections from a typical sedimentary section at normal incidence.

Reflection from first interface = 4% of incident energy; energy reflected from second interface = 1.1% of incident energy; energy reflected from deepest and highest speed bed = 0.7% of incident energy.

(j) Same as case (i) but with angle of incidence at the deepest bed = 10°.

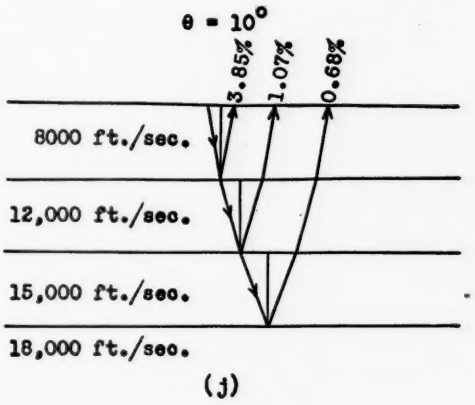
Energy reflected from first interface = 3.8% of incident energy; energy reflected from second interface = 1.1% of incident energy; energy reflected from deepest and highest speed bed = 0.7% of incident energy.

(k) The reflections at normal incidence from a typical sedimentary section with a low velocity intermediate stratum.

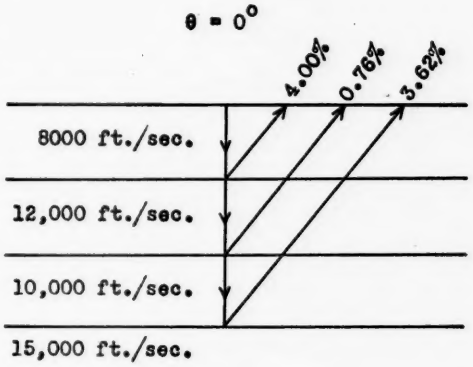
Energy reflected from first interface = 4% of incident energy; energy reflected from intermediate low velocity bed = 0.8% of incident energy; energy reflected from deepest and highest speed bed = 3.6% of incident energy.

(l) Same as case (k) but with angle of incidence at the deepest bed = 10°.

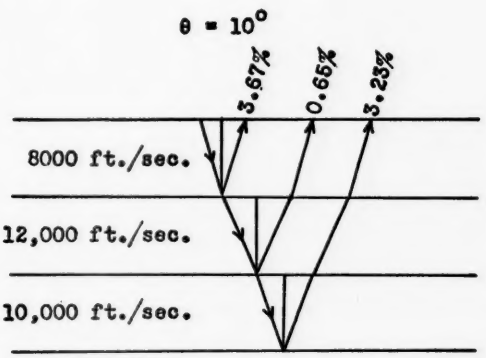
Energy reflected from first interface = 3.7% of incident energy; energy reflected from intermediate low velocity bed = 0.6% of incident energy; energy reflected from deepest and highest speed bed = 3.2% of incident energy.



(j)



(k)



(l)

A comparison of the results for cases (a), (b), and (c), or (d), (e), and (f), or (i) and (j), and the pair of cases (k) and (l), shows that while the reflected energy from the various types of interface will decrease with increasing angle of incidence the magnitude of the effect of the angle of incidence is rather small. For most cases arising in ordinary reflection shooting processes one should, therefore, be justified in basing the energy calculations upon the assumption of normal incidence. Moreover, as a comparison of cases (f) and (g) shows, the exact definition of the angle of incidence in a stratified system is of lesser importance still, and the simplest definition which refers to the angle of incidence as that at the deepest bed should suffice for all purposes when normal incidence is not explicitly assumed.

With regard to the effect of stratification, the above examples show that the intermediate interfaces will greatly attenuate the energy ultimately returning from the bed of greatest depth. Thus a single intermediate interface splitting the total velocity contrast in half will cut the fraction of the incident energy returning from the highest speed bed by a factor of about 5. Two additional interfaces will make a further reduction by a factor of about 5, so that the energy returning ultimately from the 16,000 ft./sec. velocity bed will be only 0.4% in case (h) as compared to 11.1% of the incident energy in case (a). Part of this attenuation is accounted for by the energy reflected from the intermediate interfaces. However, somewhat more than half of the total decrease is due to the higher ultimate loss of the incident energy by transmission into the highest speed bed because of the reduced velocity contrast at this bed.

Cases (i) and (j), which correspond in their gross features to the type of section encountered in certain midwestern districts, show again the high degree of attenuation resulting from the intermediate reflection processes. Under the assumed conditions less than 1% of the incident energy will return from the highest speed and deepest bed, whereas more than 5% will come back from the two upper interfaces.

The velocity sections of cases (k) and (l) show the type of effect to be expected when intermediate low velocity beds are encountered. It will be seen from these that if the section resumes its normal velocity increase with depth after passing the low velocity bed, the high contrast at the base of the latter will give rise to high reflection coefficients and greater ultimate returns of reflected energy to the surface than sections where the velocities increase monotonically with depth and all the contrasts are rather small. Thus in the specific cases of

(k) and (l), almost as much energy will be returned from the deepest bed as will from the first interface. Moreover by comparing (k) with (i) it will be seen that more than three times as much energy will be reflected from the 15,000 ft./sec. bed in case (k) than in case (i), although the latter has one less intermediate interface to be crossed by the seismic waves.

All the calculations given in the previous paper and the applications presented here are based, of course, upon the assumption that there is no absorption and attenuation of the waves within the strata themselves. These strata are assumed to be non-absorbing and ideal elastic media. This assumption will undoubtedly not be valid in the case of actual earth strata. It is nevertheless felt that the gross implications of the calculations based upon the idealized cases should remain valid under the conditions of current field practice of reflection shooting.

The authors wish to thank Dr. Paul D. Foote and Dr. E. A. Eckhardt of this laboratory for permission to publish this paper.

THE COMPUTATION AND MAPPING OF SEISMIC REFLECTION DATA*

M. B. WIDESS† AND N. A. HASKELL†

ABSTRACT

The fundamental computation equations for dip shooting are reviewed. From a consideration of the curvature of the wave front of the reflected wave an expression is derived for the angle of arrival in terms of the reflection time difference over a finite spread. The effect of curvature of the reflecting bed is discussed. An outline is given of a rigorous computational procedure which takes full consideration of the three dimensional aspect of seismic interpretation and which is based on the use of time cross-sections and maps. Since this procedure is presented on the basis of a velocity equation which is a function only of depth, it is indicated that a modification must often be made to account for the deviation of the direction of velocity gradient from the vertical.

INTRODUCTION

Until recently data in seismic prospecting had been computed generally on the basis of straight ray paths of the seismic waves because of the inherent simplicity of the computation method based on this assumption and because it usually led to sufficient accuracy for relatively shallow horizons exhibiting gentle relief. Since straight ray paths are inconsistent with the observed variation of velocity with depth, an effort is now often made to avoid the error by the use of curved ray paths in computation. Such a method succeeds in reconciling the inconsistency mentioned above but unfortunately in many cases it still exposes itself to possibly a greater error by ignoring an appreciable deviation of the velocity gradient from the vertical which in general accompanies strongly dipping strata. Nevertheless, since computation methods which assume a velocity varying only with depth are often employed, and in some cases are exactly applicable, it has been thought worth while to present some information relative to these methods which has not previously appeared in the literature. The purpose of this paper is to derive a useful dip equation applicable to the case of a velocity which is a function *only* of depth and to describe a method of reflection time mapping which is of value in seismic interpretation not only for this particular type of velocity variation but also for any other assumption as to velocity.

* Presented at the Annual Meeting, Chicago, April 9, 1940.

† Western Geophysical Co., Los Angeles, Calif.

RÉSUMÉ OF FUNDAMENTAL EQUATIONS FOR RAY PATH

In a medium in which the velocity, v, is a function only of the depth, the equation of a given ray is expressed in parametric form by[1]

$$X = \int_0^H \frac{pv\,dy}{(1 - p^2v^2)^{1/2}} \tag{1}$$

$$t = \int_0^H \frac{dy}{v(1 - p^2v^2)^{1/2}} \tag{2}$$

$$p = \frac{\sin \alpha}{v} \tag{3}$$

where X is the horizontal displacement; H is the depth; t is the travel time along the ray; α is the angle between the ray and the vertical at any point; and the parameter p is a constant for any given ray, all terms being referred to the shot-point as origin.

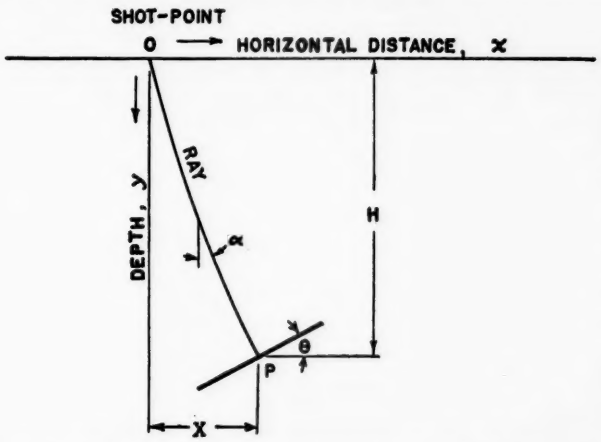


FIG. 1. Path of normal ray from shot-point to reflecting horizon.

If the ray encounters a reflecting bed dipping at an angle θ at point P, Fig. 1, and α equals θ at this point, the ray will be reflected back along its original path. Such a ray, which strikes a reflecting bed normally, is termed a "normal ray," and the reflection time, T, for a normal ray is equal to twice the time given by equation (2).

Except for certain velocity functions, the integrals of equations (1) and (2) cannot be evaluated explicitly. Where explicit evaluation

[1] See, for example, M. M. Slotnick, "On Seismic Computations," GEOPHYSICS, Vol. 1, No. 1, January 1936.

is not possible the usual procedure is either to resort to approximate integrations or to convert the integrals into series by breaking the subsurface section into horizontal laminations, each of constant velocity. It is not the purpose of the present discussion to consider the various methods by which the integrations may be accomplished, but rather to outline a method of applying the solution to field data. It will therefore be assumed that these integrals have been evaluated and the results tabulated or presented in appropriate graphical form so that the depth, dip, and displacement may be regarded as known functions of the reflection time, T, and ray constant, p, of the normal ray.

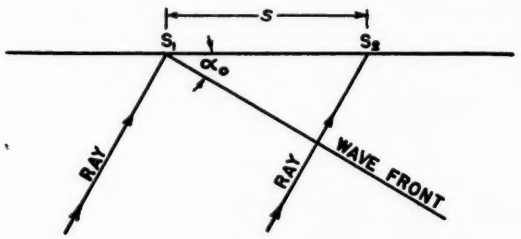


FIG. 2. Plane wave front impinging on seismometer spread.

The fundamental relation between the ray constant and the observed data may be found by reference to Fig. 2. The two seismometers S_1 and S_2 are located at the same elevation and separated by a distance s which is assumed to be small enough that the wave front striking the seismometers may be treated as a plane. This plane is further assumed to be at right angles to the plane of the paper. When the wave front is inclined at an angle α_0, it is evident that the difference ΔT between the reflection times at the two seismometers is given by

$$\Delta T = s \frac{\sin \alpha_0}{v_0} \tag{4}$$

where v_0 is the velocity at the top of the subsurface section. The reflection time difference, ΔT, ("move-out" or "step-out" as it is commonly called) therefore determines the angle of arrival of the wave.[2]

Since, from equation (3), the ratio of sin α to v at any point along a ray is equal to the ray constant, equation (4) may be written

$$\frac{\Delta T}{s} = p. \tag{5}$$

[2] It should be noted that equation (4) defines the maximum ΔT that can be recorded. That is, ΔT cannot exceed s/v_0, which is the "move-out" obtained when the ray travels parallel to the ground surface.

If the seismometers S_1 and S_2 are placed at equal distances on opposite sides of the shot-point, the angle of arrival measured by ΔT is that of the normal ray. The dip of the reflecting bed at point P, Fig. 1, is obtained by combining equation (3) and (5), namely

$$\sin \theta = \frac{\Delta T}{S} v_p, \tag{6}$$

where v_p is the velocity at point P.

Equations (1), (2), (5), and (6) are the fundamental set required for the conversion of the field data, namely T, ΔT, and s, into space variables, namely θ, H, and X at the reflecting point. However, the dip equations (5) and (6) are accurate only when the depth is considerably greater than s, since a plane wave front was assumed, and further only when the line of seismometers is oriented in the direction of dip of the reflecting strata. Because in many cases longer spreads are employed and in general the direction of dip is unknown prior to the shooting, certain appreciable inaccuracies are often introduced by the use of the given equations. In the following sections methods are developed which overcome to a great extent these limitations which are imposed on the above equations.

THE GENERAL DIP EQUATION

In order to obtain the required degree of accuracy when treating long spreads or reflections from shallow beds, a new dip equation will be derived which takes into account the fact that the wave front in the neighborhood of the seismometers is not a plane, as assumed in the preceding elementary development, but is a curved surface. In carrying the expression for the dip to the next higher order of approximation, we may consider the wave front in the neighborhood of the seismometers to be a spherical surface whose radius of curvature is roughly equal to the length of the ray path from shot-point to reflection point and back to the recording seismometer. This is exactly true if the velocity of the entire subsurface section is constant, and is generally a sufficiently close approximation for the velocity equations which are in actual use.

A relation between dip and the observed data for a curved wave front may be found by reference to Fig. 3, where are shown a shot-point at O and a seismometer at S, both at the same elevation, and separated by a distance x. The reflecting horizon is a plane normal to the plane of the paper and dipping at an angle θ. The curve W repre-

sents the wave front of the reflected wave at the instant it reaches the shot-point, where the wave impinges at an angle α_0, the angle of arrival of the normal ray. Let the center of curvature be at C; that is, to an observer at the earth's surface the wave appears to diverge from the point C. For simplicity of derivation we shall assume that the velocity is constant and equal to v_0 down to the depth of the point A, Fig. 3.

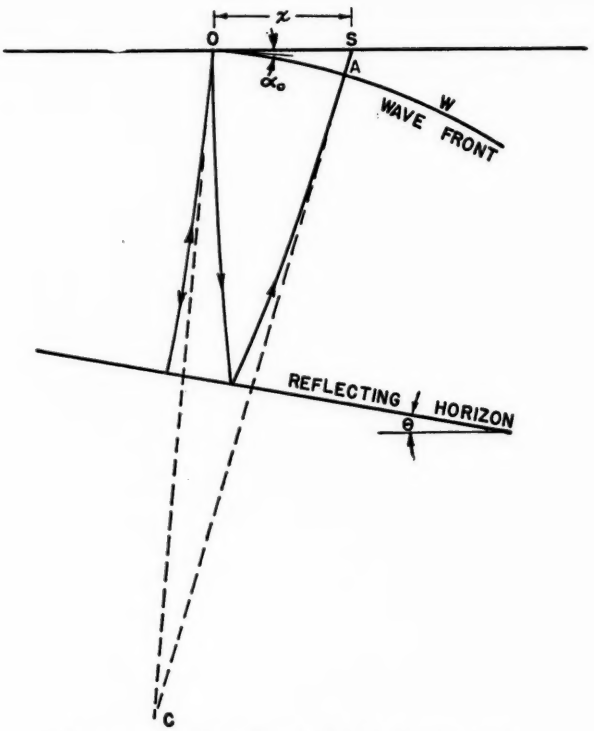


FIG. 3. Reflection rays and spherical wave front.

The difference, ΔT, between reflection times to O and S is evidently

$$\Delta T_x \equiv T_x - T_0 = \frac{\overline{AS}}{v_0} . \tag{7}$$

In the triangle OCS we have

$$\overline{CS} = (\rho^2 + x^2 + 2\rho x \sin \alpha_0)^{1/2} \tag{8}$$

where $\rho \equiv \overline{OC} = \overline{CA}$ is the radius of curvature of the wave front. Expressing the radical as a power series in (x/ρ), we obtain

$$\frac{\overline{CS}}{\rho} = 1 + \frac{x}{\rho}\sin\alpha_0 + \frac{1}{2}\left(\frac{x}{\rho}\right)^2\cos^2\alpha_0 - \frac{1}{2}\left(\frac{x}{\rho}\right)^3\sin\alpha_0\cos^2\alpha_0 + \text{terms of order } \left(\frac{x}{\rho}\right)^4 \text{ and higher,}$$

therefore, after substituting into equation (7),

$$\Delta T_x = \frac{\overline{CS} - \rho}{v_0} = \frac{\rho}{v_0}\left\{\frac{x}{\rho}\sin\alpha_0 + \frac{1}{2}\left(\frac{x}{\rho}\right)^2\cos^2\alpha_0 - \frac{1}{2}\left(\frac{x}{\rho}\right)^3\sin\alpha_0\cos^2\alpha_0 + \text{terms of order } \left(\frac{x}{\rho}\right)^4 \text{ and higher}\right\}. \tag{9}$$

This is the general equation which replaces equation (4). The reflection time difference between any two seismometers may be obtained by substituting separately the corresponding values of x into equation (9) and subtracting the resulting values of ΔT, the positive direction of x being taken down dip. In particular, for a symmetrical split spread (i.e., seismometers colinear and symmetric with respect to the shot-point) of length s, we have[3]

$$\Delta T = \Delta T_{s/2} - \Delta T_{-s/2}$$
$$\frac{\Delta T}{s} = \frac{\sin\alpha_0}{v_0}\left\{1 - \frac{1}{8}\left(\frac{s}{\rho}\right)^2\cos^2\alpha_0 + \text{terms of order } \left(\frac{s}{\rho}\right)^4 \text{ and higher}\right\}. \tag{10}$$

The terms of order $(s/\rho)^4$ and higher are negligible in almost all practical cases and, therefore, could be dropped. Introducing the spread length correction factor γ defined by

[3] It can be shown that, due to the symmetry of the spread, the assumption of a constant velocity to the depth A, Fig. 3, is not a necessary condition for the validity of equation (10) for the velocity functions generally used. The velocity v_0 in equation (10) should, therefore, be the velocity at the top of the section.

$$\gamma = 1 - \frac{1}{8}\left(\frac{s}{\rho}\right)^2 \cos^2 \alpha_0 + \text{terms of order } \left(\frac{s}{\rho}\right)^4 \text{ and higher, (11)}$$

we may write equation (10) in the form

$$\frac{\Delta T}{s} = \gamma \frac{\sin \alpha_0}{v_0} . \tag{12}$$

Since for an infinitesimal spread $\Delta T/s$ is the derivative of reflection time with respect to lateral position, and equation (4) applies when the spread is very short, equation (12) can be written in the form

$$\left(\frac{\Delta T}{s}\right) \text{ finite split spread} = \gamma \left(\frac{dT}{ds}\right) \text{ at the shot-point.} \tag{13}$$

Thus the consideration of a curved wave front yields, in place of equation (5), the following more accurate expression for the normal ray constant

$$p = \frac{1}{\gamma}\left(\frac{\Delta T}{s}\right) \text{ finite split spread.} \tag{14}$$

The computation of γ requires an evaluation of the radius of curvature ρ, a laborious process if carried out rigorously. Fortunately, however, great accuracy is not required since ρ appears in a relatively small term in the dip equation, and it is generally sufficient to use the product of the reflection time and the average velocity to the reflecting point.

Besides indicating the relation between the observed ΔT and the reflection time differential which is applicable to the fundamental computation equations, equation (13) will also serve to determine the reflection time gradient useful in the preparation of the time maps which will be outlined in a later section.

EFFECT OF CURVATURE OF THE REFLECTING HORIZON

The derivation of the spread-length correction γ was based on a reflecting horizon which is a plane. It might be conceived, however, that the effect of curvature of the reflecting horizon on the accuracy of the computation equations might be significant. Nevertheless, the following consideration shows that in general this effect can be ignored. It is the order of magnitude of this effect rather than a method of determining the curvature which will be evaluated here,

and accordingly it will be considered that the results may be sufficiently approximated by treating a case of constant velocity throughout the medium.

It is clear that a particular horizon can be so severely distorted that for a single shot-point a seismometer may respond to reflections from several different points on that single horizon. Such a condition of multiple reflecting points will not be considered at present, but rather the simple case in which there exists a one-to-one correspondence between seismometer position and point of reflection.

A well known law of geometric optics states that when a wave is reflected from a curved surface, the radii of curvature of the incident and reflected wave fronts, ρ_i and ρ_r respectively, are related to the radius of curvature ρ_s of the reflecting surface by the equation

$$\frac{1}{\rho_i} + \frac{1}{\rho_r} = \frac{2}{\rho_s}.$$

In using this equation, convex upward curvature will be called positive and convex downward negative. Letting the path length from the shot-point to the point of reflection be $l/2$, the radius ρ_i is given by $-l/2$. The reflected wave therefore starts back with a radius of curvature given by

$$\rho_r = \frac{1}{2}\,\frac{\rho_s l}{\rho_s + l}.$$

When the wave arrives back at the shot-point, the radius of curvature is

$$\rho = \frac{l}{2} + \rho_r = l\,\frac{1 + \dfrac{l}{2\rho_s}}{1 + \dfrac{l}{\rho_s}}. \tag{15}$$

Therefore, in order of magnitude, the effect of curvature of the reflecting surface amounts to changing the curvature of the reflected wave front from l to the value given by this equation. Since in general the effect of the wave front curvature term in the computation equations is small for a plane reflecting bed, the change due to curvature of the reflecting horizon will also usually be small. In rare cases, because of curvature of the reflecting horizon, the value of ρ may be so small as to produce a marked effect in γ, but in these instances the

reflecting area on a particular bed becomes so broad that generally the resultant wave front assumes a complex and far from spherical form. In particular cases where multiple reflecting points occur, the various branches of the complex wave front will interfere if they arrive nearly simultaneously, while, if they do not interfere, each branch can be treated as a separate reflection in the manner described.

RIGOROUS TREATMENT OF THREE DIMENSIONS

In the derivation of equations in preceding sections it has been assumed that the seismometer spread is oriented along the direction of maximum dip of the reflecting horizon, a direction which of course is seldom known in advance. The problem of determining the dip from data derived from obliquely oriented spreads is thus introduced. There is, however, the even greater problem of linking data from adjacent shot-points; that is, even though isolated reflection points and dips may be determined from individual shot-points, how is this information to be assembled to form a map of some particular horizon, especially in the absence of correlatable reflections? Since points of reflection for seismometers and shot-points all of which are oriented along a straight line are not necessarily themselves on a straight line, no single vertical plane cross-section can be drawn through these points to represent the observed data. A laborious method involving segmented cross-sections, or an equivalent procedure, must be used. In fact, it becomes evident that a method dealing with reflection points after each is determined individually is necessarily inapplicable to the drawing of profiles along a straight section and, more important, that rigorous handling of data so obtained cannot be simple. The problem confronting the interpreter is, therefore, one which involves not only the proper computation of data at each shot-point but also the organization and integration of this data from all shot-points.

The situation is commonly handled approximately, by applying the observed data to the equations which are valid for spreads oriented along maximum dip and by treating the resulting computed angle as a component of dip in the direction of the spread. This method is generally sufficiently accurate for horizons of gentle relief, but in other cases a correction is sometimes desired.

A procedure which will achieve solution of the stated problem and one which will retain relative simplicity can be evolved by introducing the concept of time maps. Though time maps have been used in the past, and in fact were employed at the inception of seismic

reflection prospecting, they were not used to accomplish a rigorous treatment of the three dimensions involved.

The three-dimensional problem may be treated as follows. From every point on a continuous reflecting horizon whose slope is also continuous a normal ray is conceived to be drawn to the ground surface. If the curvature of the reflecting horizon is not so severe as to cause some of the rays to cross, a one-to-one correspondence is established in this manner between points on the reflecting horizon and points on the ground surface. Consequently, when the reflection time along normal rays is known for all points on the ground surface, a

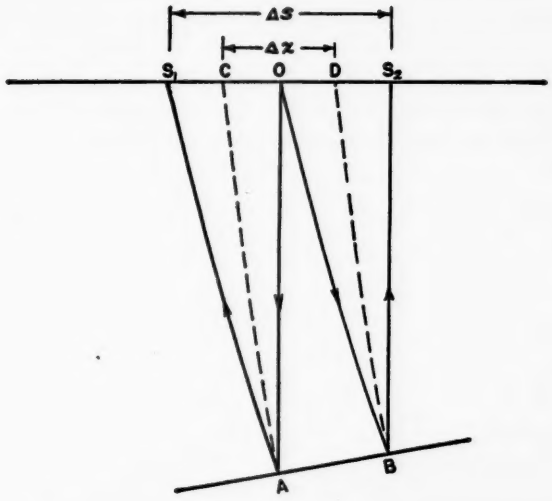


FIG. 4. Reflection rays to seismometers compared with normal rays.

unique reflecting horizon is determined for any given velocity function. A plot of reflection times along normal rays for a particular reflecting horizon will be called a "reflection time map."

It is necessary that at each shot-point a determination be made of the reflection time derivative, dT/ds, as well as of the reflection time itself before an exact evaluation of the position of the reflection point from that shot-point is possible. This information is obtained directly from observed data in dip shooting and is to be used in the preparation of reflection time maps. The reflection time derivative must be transformed to a gradient of the normal ray reflection time, i.e., the change of reflection time along normal rays with respect to

change of position of the point on the ground surface at which the normal rays terminate. Fig. 4 shows a shot-point at O and an infinitesmal symmetrical spread, Δs. From reflecting points A and B normal rays are drawn, intersecting the ground surface at points C and D, which are separated by Δx. It is evident that the spread length Δs is equal to twice Δx in the limit as Δs approaches zero, i.e.,

$$\frac{ds}{dx} = 2.$$

The normal ray reflection time gradient, which will be called simply "time gradient," is thus given by

$$\frac{dT}{dx} = 2\frac{dT}{ds}. \tag{16}$$

Since we have already obtained the relation between dT/ds and the observed reflection time difference over a finite spread, the reflection time and its gradient are thus obtained at every point wherever dip shooting[4] is employed.

When no dip data has been obtained and correlation can be effected, a time map is essential to a rigorous interpretation of the data since the position of the reflecting point cannot be determined from a single reflection time alone. In this case preparation of the reflection time map is simple in that the observed reflection times for any correlatable horizon are plotted at the shot-points and lines of constant reflection time, or *iso-times*, are drawn from this data. In determining the position of the reflecting horizon, the ray constant $p=dT/ds = \frac{1}{2}(dT/dx)$ may be obtained from the time map.

Where correlations are not available and dip phantoms would normally be run, the time gradients obtained from dip shooting according to equations (13) and (16) are used in running reflection time phantoms. This involves no approximation, as does the customary projection of dip phantoms, since the time gradient obtained from any spread is the true gradient in the direction of that spread regardless of how it may be oriented with respect to the dip of the reflecting horizon. In running the time phantom a starting reflection time is determined and this time is carried along a straight line from one

[4] Symmetrical split spreads in particular have been considered in the present discussion. Any other type of spread can be readily adapted to a similar treatment by the use of the basic dip equation (9).

shot-point to another by using at each shot-point the time gradient which was determined in the direction of that line. The time phantom of the first line is carried into other lines at tie points, and the errors of closure in closing traverses are adjusted in the usual manner. The times along this phantom are then transferred to a map and iso-time contours drawn. When spreads at right angles to shot-point lines are also used at some shot-points, the time gradient in the cross direction is compounded, in the manner of components of a vector, with the gradient in the line direction. The resultant time gradient vector is then drawn at the shot-point to assist in the contouring.[5]

The time map is then transformed to a final depth map by the application of equations (1), (2), (14), and (6), the last being now written

$$\frac{dT}{dx} = \frac{2 \sin \theta}{v_p} . \tag{17}$$

These equations are valid only along the direction of maximum dip, and, therefore, the gradient dT/dx, must be the maximum gradient. The procedure in converting from time to depth is, therefore, to choose points distributed in some manner over the time map, to read off the reflection time and measure the direction and magnitude of the maximum time gradient from the time map at these points, and to compute the space coordinates of the corresponding reflecting points by using computation charts prepared on the basis of the above equations. The computed horizontal displacement is then plotted up-dip in the direction of maximum gradient at the points of computation, and the depths are recorded at the reflection points so determined. Contours for the map are finally drawn on the basis of these depths.

It is observed that all useful data obtained from dip shooting have been employed and, most important, they have been integrated by the reflection time map in the proper relation with respect to each other. No errors have been incurred, or even consideration required, by the three dimensional aspect of the problem. This was made possible because the "raw" data, being referred to the location where the data are obtained, has not introduced any unknowns during the process of integrating the information such as would be the case if the observed

[5] It has been assumed that the condition of multiple reflecting points was not present; that is, the reflecting horizon was not so severely distorted that several points on the reflecting horizon were joined by normal rays to a single point on the ground surface. When multiple reflecting points are encountered, the time map consists in part of an odd number of multiple branches, which complicate the problem.

set of data at each shot-point were individually transformed to space coordinates. Furthermore, the operations of transforming time data to space variables which are accurately applicable only for spreads along the direction of maximum dip can be employed rigorously.

We have shown how a rigorous and relatively simple method can be employed for computation of seismic data; however, this method cannot insure that serious errors in completed maps will not be introduced because of an incorrect assumption as to the velocity. It has been indicated that the outlined method was derived on the basis of a velocity equation which is a function only of depth, so that, while time mapping is applicable for any type of velocity variation, the computed depth may be in considerable error when the velocity is no longer a function only of depth. Indeed in the special case of an entire shallow section consisting of parallel strongly dipping strata, it would probably be more accurate to assume a straight ray path normal to the beds rather than to assume a curved ray path in a medium which is considered to possess no deviation of the direction of the velocity gradient from the vertical. If the latter procedure were followed, one would generally compute dips which are greater than the actual existing dips. Though the true velocity variation can be known only in regions where wells which have been shot for velocity are densely spaced, still it is evident that the local deviation of velocity gradient from the vertical can in general be considered to be dependent on the degree of dip of the strata. Consequently it appears important to introduce a dependence of this type into curved path computations, which factor will partially correct, at least in the proper direction, for the effect of dipping strata on velocity variation. Such a procedure has already been put into practice.

CHANGE OF REFLECTION AMPLITUDE AND CHARACTER WITH GEOPHONE DEPTH*

H. M. HOUGHTON†

ABSTRACT

The relation between character and amplitude of reflection and the geophone depth was investigated at several locations in Oklahoma. It is shown that the variation of reflection amplitude down to a depth of the order of 200 feet is, in general, what would be expected from changes of the acoustic impedance of the strata traversed. Changes in reflection character are illustrated by tracings from the records obtained.

This paper describes some experimental work done to ascertain whether the depth at which a geophone is placed beneath the surface has an appreciable effect upon the character and amplitude of the recorded reflections. The tests were made at four different locations, two in Pottawotamie County, and two in Logan County, Oklahoma. Both locations are in areas where good reflections are obtained with comparatively small charges, and where shot holes stand up long enough so that many successive shots can be fired without much change in the character of the excitation.

The method employed was simply that of making a comparison of the response of a geophone placed at a depth of two feet below the surface with that of a geophone planted at the bottom of an adjacent hole drilled to various depths. A driller's log was obtained while the deep holes were drilled. For convenience, the geophone which was placed at the two foot depth will be referred to as the "check" geophone; the geophone at varying depths, as the "test" geophone. At the beginning of each series of records, the test geophone was placed in a two foot hole beside the check geophone and the two recording channels were adjusted to give the same record amplitude. The test geophone was then placed at the bottom of the drilled hole. Ample slack was left in the cables so that they transmitted no appreciable energy down from the surface. One or more records were then taken, the charge being varied when necessary to control the amplitude of the check trace. After removing the test geophone, a small charge was fired at the bottom of the test hole and was recorded on the check geophone to determine the amount of time "lead" to be expected on

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the test trace. This time was quite helpful in identifying corresponding points on the records, and the data thus obtained were also used to compute the velocity-depth relationship in the material traversed by the holes. The test hole was then deepened, and the above procedure was repeated. Near the surface, successive test geophone positions were only few feet apart, while below 20 feet the holes were

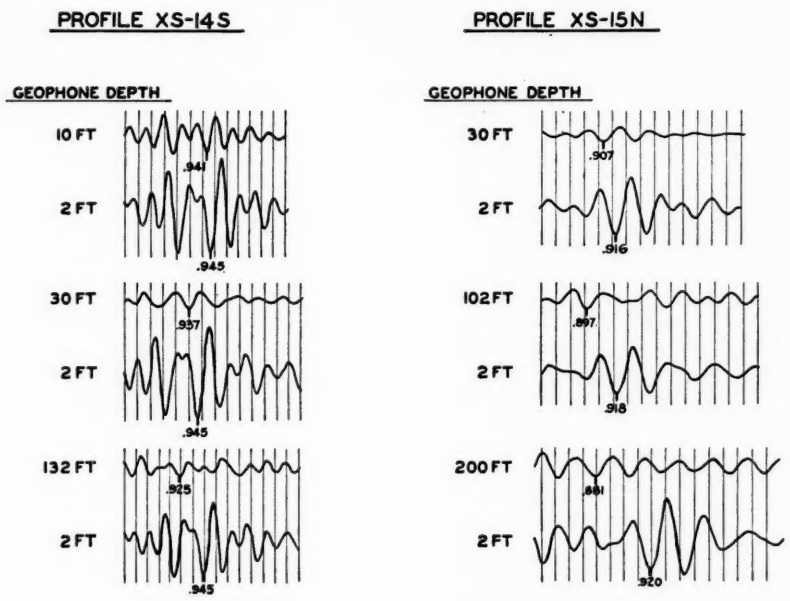

FIG. 1. Tracings illustrating changes in reflection character.

deepened in steps of 20 to 60 feet. The total depth of the holes used varied from 160 to 240 feet.

In the first two holes investigated, it was found that the decrease in amplitude with depth on the test trace was frequently so great that it was difficult to compare the character of the test trace with that of the check trace. Therefore, in the second half of the project, after taking records with the same sensitivity for amplitude comparison, the gain of the test channel was increased by a known amount when necessary, in order to make the two amplitudes similar.

The resulting records were examined, first, to compare the character of the reflections as recorded by the test geophone in the deep hole with that recorded by the check geophone, and second, to deter-

mine the ratio of the amplitude of a selected reflection as recorded by the two geophones.

The first location occupied was in Pottawatomie County, and is designated XS-14. The character of the recorded reflection is shown in Fig. 1. It will be noted that the character of the test trace for a geophone depth of 10 feet is very similar to that recorded by the check geophone. The amplitude of the test trace is somewhat smaller, and

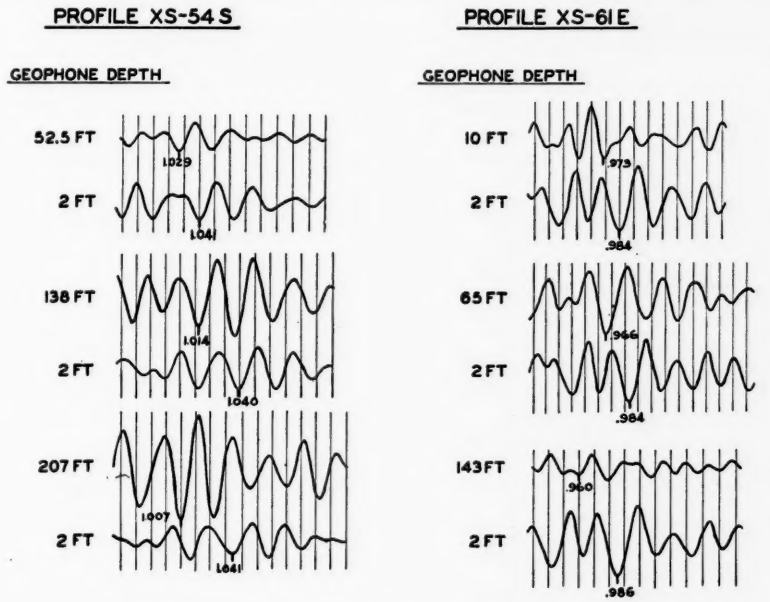

FIG. 2. Tracings illustrating changes in reflection character.

the reflection arrives .004 second earlier than on the check trace. At 30 feet the amplitude has continued to decrease, and the amplitude of the peak immediately following the marked trough has decreased relative to adjacent legs. The record taken in the 132 foot hole shows still further changes in character.

Records taken at the second location in Pottawatomie County, XS-15, are also illustrated in Fig. 1. These show similar changes in character with an apparent decrease in prominence of the reflection. This effect is partly due to the decreased amplitude.

Location XS-54 was in Logan County, Oklahoma. The amplitude of the traces here shown (Fig. 2) can not be compared directly, because

the gain of the recording channel fed by the test geophone was increased. This series of records shows no appreciable change of character of reflection with geophone depth.

The records at location XS-61, also in Logan County, showed appreciable character changes with depth, some of which are illustrated by their tracings in Fig. 2.

The variation of the amplitude of a reflection with geophone depth, and also a velocity-depth graph and a driller's log of each hole are shown in Figs. 3–6.

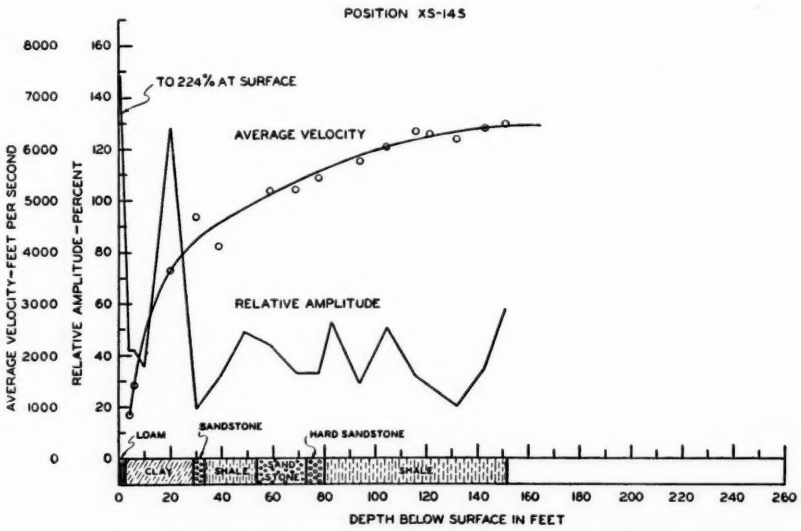

FIG. 3. Variation of reflection amplitude with depth.

At position XS-14, the velocity-depth graph shows an average velocity of approximately 900 ft. per second within a few feet of the surface, but this increases rapidly to 6500 ft. per second at a depth of 150 feet. The reflection amplitude is greater with the geophone at the surface than at a depth of 2 feet; it decreases when the depth is ten feet, and shows an increase at a depth of 20 feet. Below this depth, the amplitude shows but little variation and in general remains at approximately 30 to 40% of the amplitude of the check geophone.

The amplitude-depth graph at location XS-15 shows a maximum at the surface, and a rapid decrease within a few feet to a value of about 20% of the amplitude at the two foot depth. At this location,

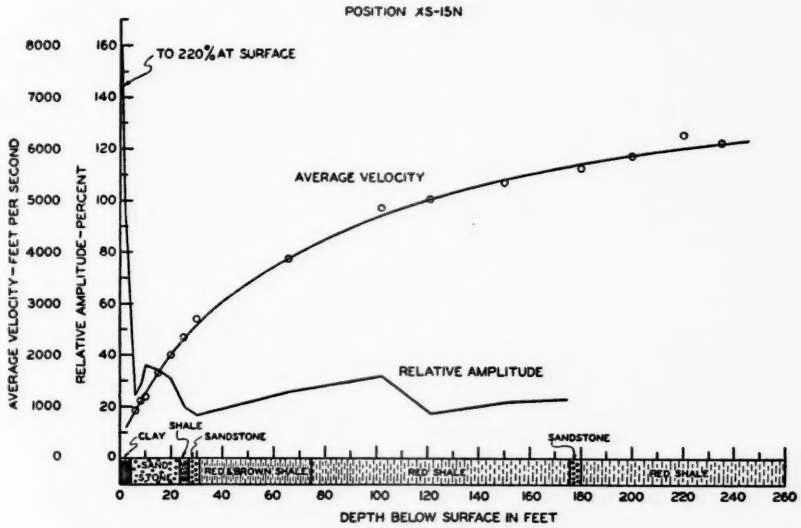

FIG. 4. Variation of reflection amplitude with depth.

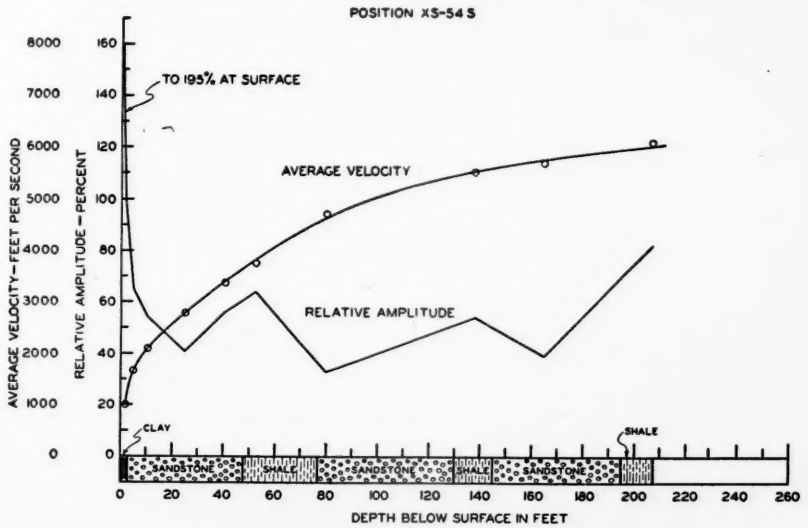

FIG. 5. Variation of reflection amplitude with depth.

also, the maximum amplitude is observed at the surface; it decreases rapidly in the first 20 feet to about 40% of its magnitude at 2 feet, and, except for minor variations, remains at about that level as the depth is further increased.

With the exception of a peak at a depth of 200 feet, the amplitude graph for Profile XS-61 is similar to the preceding graphs. Here again there is a very rapid decrease in amplitude within the first 20 feet.

As in most experiments involving the materials of the earth, no complete explanation can be given for all the phenomena observed. However, one characteristic common to all the amplitude-depth relationships, namely, a rapid decrease in reflection amplitude with a moderate increase in depth, is undoubtedly the result of an increase in acoustic impedance with depth. The velocity increases rapidly with depth and consequently also the acoustic impedance, (velocity times

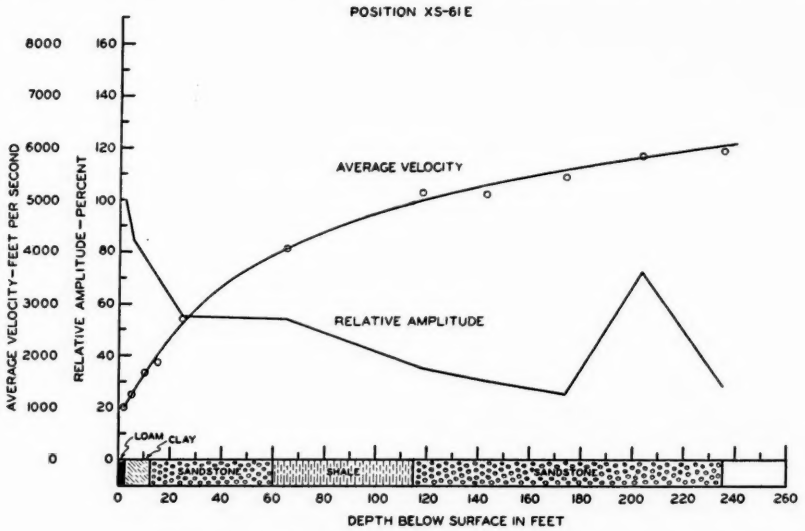

FIG. 6. Variation of reflection amplitude with depth.

density), since in general the density remains fairly constant or increases slowly with depth. The amplitude of a sound wave traversing a medium in which the acoustic impedance varies, will be inversely proportional to the acoustic impedance. It follows that the amplitude of a wave approaching the surface will increase as its velocity decreases. The presence of layers of various materials near the surface complicates matters. Each interface may act as a reflecting surface, and so alter the character and amplitude of the reflection observed at various depths.

An effort was made to correlate the amplitude and character of the reflections obtained with the type of material in which the geo-

phone was planted at the various depths, but no such correlation was evident.

An attempt was also made to see if the character variations observed could be completely explained by the assumption that the motion of the geophone at any depth was simply the result of the movement caused by waves arriving from below plus the reflection of those waves from the earth's surface. This study was made by graphically combining, in the appropriate phase relationship, two wave trains having the shape recorded by a geophone at the surface. This work was not successful; it is apparent that other factors are involved.

In conclusion, one other point may be of interest. It was found that, in general, no appreciable gain in the ratio of reflection amplitude to ground unrest was obtained as the geophone was lowered below the surface. The decrease in ground unrest was approximately the same as the decrease in reflection amplitude observed, so that the ratio remained nearly constant. This relationship may not hold in other areas, or at greater depths, but our observations would indicate that no great change in the signal-noise ratio will result if the geophone is planted in deep holes.

RELATIVE GRAVITY MEASUREMENTS USING PRECISION PENDULUM EQUIPMENT*

MALCOLM W. GAY†

ABSTRACT

The precision with which relative-gravity values may be obtained by the use of suitable pendulum equipment is illustrated by actual data obtained in establishing calibration stations used to facilitate the calibration of gravimeters.

A résumé of the operating technique, together with the salient features of the Gulf pendulum equipment is given in connection with a tabulated summary of the various factors which must be considered in designing and operating pendulums in order to attain the desired precision of 10^{-7} g.

INTRODUCTION

A convenient way to calibrate a gravity meter is to operate it at two locations between which the difference in the acceleration of gravity is known. It is the purpose here to present data and values, obtained by pendulum observations, to illustrate the precision with which it is possible to measure such gravity differences. Pendulum observations made in 1935 and 1936 on two pairs of locations near the laboratory of the Gulf Research & Development Company[1] furnish, as far as is known, the most precise measurements of this kind that have been made.

PENDULUM APPARATUS

Pendulum apparatus specifically designed for field service was used to make the measurements. The pendulums (Fig. 1) are ground[2] from a single piece of fused quartz[3] except for the fused quartz plugs that form the knife-edges. These plugs are cemented in a hole in the head of the pendulum. Massive blocks of Pyrex glass[4] (Fig. 1) constitute the bearings on which the knife-edges ride when the pendulums are swinging. The narrow sides of the heads of the pendulums are optically flat and are coated with evaporated aluminum. A beam of light reflected from these mirror surfaces traces a record of the

* Presented at the Annual Meeting, Chicago, April 9, 1940.

† Gulf Research & Development Company.

[1] Located at Harmarville, near Pittsburgh, Pennsylvania.

[2] Mr. J. W. Fecker of Pittsburgh ground the pendulums from the fused quartz blanks.

[3] The fused quartz blanks were made by the General Electric Company.

[4] Made by the Corning Glass Works. Optical work was done by Bausch and Lomb Optical Company.

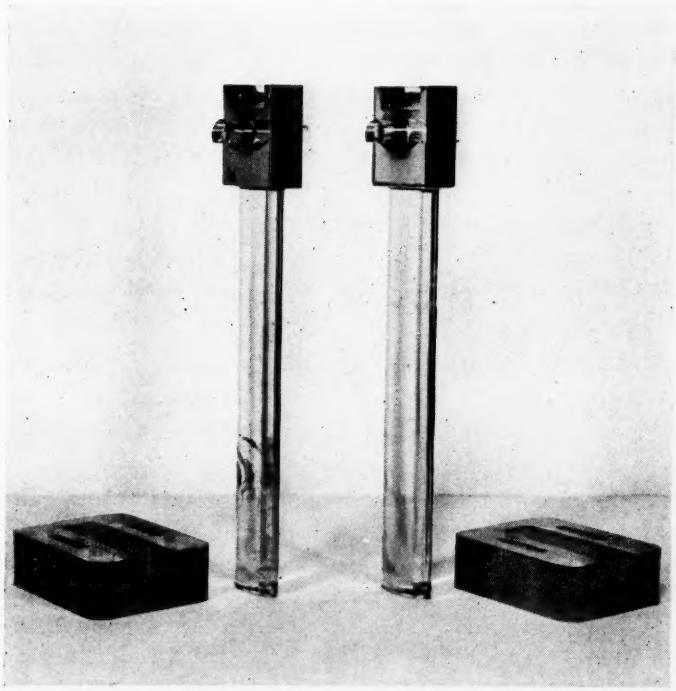

FIG. 1. A matched pair of fused quartz pendulums and their Pyrex glass bearings. The periods of these two pendulums are alike to within 0.000,002 second. This period is, very roughly, 0.89 second. The pendulums are approximately 30 cms. long and weigh about 590 grams. A field instrument, or unit, includes two pendulums and their bearings in an aluminum alloy case where they are mounted on a common support and from which they swing 180° apart in phase.

pendulum motion upon a moving photographic tape. The combination of pendulum and bearing is installed in an aluminum-alloy case. One instrument includes two such combinations mounted on the same support from which the pendulums are swung 180° apart in phase.[5] The photographic tape operates in a camera mounted on a separate recording unit.

DISTURBANCES TO THE PENDULUM PERIOD

For high precision it is more desirable to eliminate extraneous effects that tend to disturb the pendulum period than to make elabo-

[5] See "Observations De Pendule Dans Les Pays Bas" by F. A. Vening-Meinesz, p. 12 et seq. Publication de la Commission Geodesique Neerlandaise, 1923.

rate corrections for them when reducing the observations. An effect that can be held constant is automatically eliminated from the reductions when relative gravity measurements are made. Accordingly, some of the interesting features of this operation are the means taken to eliminate, or to hold constant, the several effects that are known to disturb the pendulum period. Table 1 presents a list of these effects and the methods used in controlling them. For the land operation of pendulums this list of effects is quite complete. Many details of the design and construction of the instruments can be inferred from the data and remarks contained in the second and third columns of the table.

TWO METHODS FOR MAKING RELATIVE GRAVITY MEASUREMENTS

A relative gravity value can be obtained through the formation of certain ratios of the total numbers of oscillations observed by means of two pendulum instruments. These are operated during two or more observations such that in any observation the total number of oscillations for each instrument occurs within identical periods of time. Such ratios can be formed in two ways and each method of forming them implies a definite field procedure.

FICTITIOUS PENDULUM

First, consider the average or "fictitious" pendulum for the matched pair swinging from the single support in any instrument. It is this fictitious pendulum that is free of certain of the disturbing factors that otherwise would affect the motion of a single pendulum, notably, the sway of the support. If

Δt = the time duration of the observation
g = the acceleration of gravity
l = the length of the equivalent simple pendulum

the total number of oscillations (swings) for one pendulum of a pair in any instrument is given by

$$s_1 = (\Delta t/2\pi)(g/l_1)^{1/2}$$

and for the other pendulum by

$$s_2 = (\Delta t/2\pi)(g/l_2)^{1/2}{}^{(6)}.$$

[6] The ratio s_1/s_2 is a measure of the adjustment or of the matching of a pair of pendulums. For equality of period this ratio is unity; practically it is usually more or less than this by a small amount. In the absence of uncompensated horizontal seismic effects, the value measures the constant difference in phase of the two pendulums, sometimes designated by μ. Variations in the μ from observation to observation indicate the presence of uncompensated horizontal seismic accelerations during the runs.

TABLE 1

Effect or Disturbance to the Pendulum Period	*Measures Taken to Eliminate or to Control the Effect*	*Remarks*
1. Unstable period resulting from an inherent instability of the material from which the pendulum is made.	1. The pendulum material is fused quartz.	1. Practical elimination of an unstable period is achieved through the use of quartz, one of the most stable materials known.
2. Effect of earth (and other) magnetism.	2. The pendulum material is fused quartz.	2. Quartz is one of the best of the non-magnetic materials. No magnetic moment has been observed in any pendulum of such magnitude as to affect its period by a measurable amount.
3. Effect due to change in the temperature of the pendulum.	3. The pendulum material is fused quartz. Pendulum case temperatures are controlled to 0.1°C at about 10°C above the average ambient temperature for the season during which observations are made.	3. Quartz possesses a low coefficient of thermal expansion. This procedure virtually eliminates the need for a temperature correction in the reduction of observations.
4. The hydrodynamic and buoyancy disturbances to the pendulum period. Effect due to changes in pressure in the atmosphere within the instrument.	4. The atmosphere within the pendulum case is evacuated to a pressure of approximately 1 mm. Hg and is there held constant.	4. The hydrodynamic and buoyancy effects are by this means minimized and held constant. Tight aluminum castings enable the maintainence of low pressures without appreciable change over periods of months.
Effects due to the presence of water vapor in the pendulum case.	The residual atmosphere in the pendulum case is dried by the use of phosphoric anhydride acid P_2O_5.	The pressure of water vapor in the pendulum case is reduced to a low value by this means.
5. Effect of the presence of electrostatic charges upon the pendulums.	5. The residual atmosphere in the pendulum case is ionized due to the presence of a radium salt whereby the charges are conveyed to the walls of the metal case.	5. This was demonstrated by experiment to be effective in the control of this disturbing factor.
6. Effect of sway (reaction of the support to the motion of the pendulum).	6. The Meinesz method of swinging two dynamically matched pendulums, 180° apart in phase, from the same support is used.	6. A typical value for the difference in periods of a pair of pendulums in a case is 4×10^{-6} second.

TABLE 1—*Continued*

Effect or Disturbance to the Pendulum Period	*Measures Taken to Eliminate or to Control the Effect*	*Remarks*
7. Effect due to horizontal seismic accelerations.	7. The Meinesz method is used. The pendulum knife-edges lie in the same horizontal plane and the pendulums execute their oscillations in opposite phase and in the same vertical plane (or in closely adjacent parallel vertical planes).	7. The average period of the two pendulums (the so-called "fictitious" pendulum) is used when the observations are reduced. Random horizontal seismic accelerations are by this means eliminated. If the distribution of these accelerations is not random, the effect on the pendulum period is averaged.
8. Effect due to changes in level.	8. The instrument is leveled to within 10–12 seconds of arc crosswise and to within 5 seconds of arc lengthwise.	8. Every precaution is taken to secure a reasonably rigid base for each set-up. If levels are known to change during an observation, that observation is discarded.
9. Effect due to finite amplitude. (Reduction to zero amplitude.)	9. Starting mechanisms are so devised that pendulums are started time after time with identical amplitudes. The pairs of pendulums are dynamically matched. The length of all observations is made the same.	9. For observations of equal duration, with the same initial amplitude, and with equal damping rates, the amplitude effect is constant and can be omitted from the reductions without affecting the precision of the result.
10. Effects due to changing knife-edge conditions: Displacement of the knife-edge along the pendulum axis (plug shift).	10. The pendulums are designed as minimum pendulums, i.e., the distance from the knife-edge to the center of gravity of the pendulum is equal to its radius of gyration about an axis through the center of mass of the pendulum.	10. The effect of a vertical displacement of the knife-edge on the pendulum period under the condition of this design is a minimum.
Fracture of the knife-edge.	Elaborate lowering and lifting mechanisms are used for getting the pendulums on and off the bearings. Handles on the instruments are interlocked with the lifting and lowering mechanism as a safety measure.	When operating the instruments some skill on the part of the operator is necessary in addition to the mechanical devices provided. The pendulum case cannot be picked up inadvertently while the pendulums rest on their bearings, because of the safety device provided.

TABLE 1—*Continued*

Effect or Disturbance to the Pendulum Period	*Measures Taken to Eliminate or to Control the Effect*	*Remarks*
Wearing of the knife-edge.	Pendulums are not placed in field service until they have been artificially aged by at least 100 hours of swinging.	The pendulum period stability is enhanced by this procedure.
Skidding the knife-edge when the pendulums are started or stopped.	The starting mechanisms are so constructed that bumper arms touch the pendulums at their centers of percussion.	Motion thus imparted to the pendulum consists of pure rotation, without translation.
11. Effects due to changing bearing conditions. Worn bearings.	11. Bearings are made from massive blocks of Pyrex glass, thoroughly annealed, ground, and polished flat to $\frac{1}{4}$ wave length sodium light.	11. These blocks never showed any signs of wear and they never warped. They met admirably the two conditions desired in a pendulum bearing, namely, that they remain flat and coplanar.
12. Effects due to vertical seismic accelerations.	12. These can be eliminated (or averaged) by using four pendulums swinging in pairs with certain definite phase relationships.	12. For most land observations the effect of these is small enough to be ignored.
13. Effect of the mutual attraction of two pendulums in close proximity in a case.		13. This gravitational effect is so small that it can be ignored.

The average pendulum of the pair in this instrument, i.e., the fictitious pendulum, makes

$$S = (s_1 + s_2)/2 = \Delta t K g^{1/2} \qquad (1)$$

total oscillations. Here

$$K = 1/4\pi\left[(1/l_1)^{1/2} + (1/l_2)^{1/2}\right]$$

and is called the instrument constant for the particular pair of pendulums or instrument. Hereafter in this paper, when the total number of oscillations or swings of a pendulum is mentioned, it is to this average of fictitious pendulum that such reference is made unless specified to the contrary. The foregoing description applies to a single instrument; to make a relative gravity measurement with pendulums requires two such instruments and at least two observations or "runs" must be taken with both instruments operating simultaneously.[7]

[7] The use of crystal controlled clocks as time standards would modify this procedure.

FIRST METHOD FOR COMBINING THE TOTAL NUMBERS OF SWINGS

Consider two pendulum instruments operated simultaneously during a time Δt_1 at a place where the acceleration of gravity is g_1. K_1 and K_2 are the instrument constants of the two pairs of pendulums respectively. Each instrument will execute a total number of swings as follows

$$S_1 = \Delta t_1 K_1 g_1^{1/2}$$

$$S_2 = \Delta t_1 K_2 g_1^{1/2}.$$

Form the ratio

$$S_2/S_1 = K_2/K_1. \tag{2}$$

Now the instrument with subscript (2) is moved to a place where the acceleration of gravity has a value g_2 and the two instruments again are operated simultaneously[8] during a time Δt_2, (Δt_2 is commensurate in magnitude with Δt_1, not necessarily equal but convenient if nearly so). For this new situation the total swings for each instrument will be

$$S_1' = \Delta t_2 K_1 g_1^{1/2}$$

$$S_2' = \Delta t_2 K_2 g_2^{1/2}.$$

Again form a ratio

$$S_2'/S_1' = (K_2/K_1)(g_2^{1/2}/g_1^{1/2}). \tag{3}$$

Both ratio equations contain the instrument constants. These can be eliminated if the ratio of equation (3) to equation (2) is formed.

$$R = (S_2'/S_1')/(S_2/S_1) = g_2^{1/2}/g_1^{1/2}.$$

Squaring, subtracting 1 from both sides of the equation, and setting the gravity difference $(g_2 - g_1) = \Delta g$, obtain

$$\Delta g = g_1[R^2 - 1]. \tag{4}$$

From this it is apparent that two observations suffice to obtain a relative gravity value. However, in practice it is customary to return the instrument with subscript (2) to the location where the acceleration of gravity is g_1 and there make a third observation operating both

[8] Arranged by means of radio communication.

instruments. This procedure standardizes the instruments before and following the field observation. In Fig. 2, observations 1, 2, and 6 furnish the standard curve to which the points of observations 3, 4, and 5 are referred.

SECOND METHOD FOR COMBINING THE TOTAL NUMBER OF SWINGS

Suppose the instrument with subscript (1) occupies a location where the acceleration of gravity is g_1, the instrument with subscript (2) occupies a location where the acceleration of gravity is g_2, and both instruments are operated simultaneously during a time Δt_1. Total numbers of swings are given by

$$S_1 = \Delta t_1 K_1 g_1^{1/2}$$

$$S_2 = \Delta t_1 K_2 g_2^{1/2}.$$

Form the ratio

$$S_2/S_1 = (K_2/K_1)(g_2^{1/2}/g_1^{1/2}). \tag{5}$$

Now let the instruments exchange locations. For this new situation the total numbers of swings are

$$S_1' = \Delta t_2 K_1 g_2^{1/2}$$

$$S_2' = \Delta t_2 K_2 g_1^{1/2}.$$

Again form a ratio

$$S_2'/S_1' = (K_2/K_1)(g_1^{1/2}/g_2^{1/2}). \tag{6}$$

The instrument constants can be eliminated by taking the ratio of equation (5) to equation (6).

$$R = (S_2/S_1)/(S_2'/S_1') = g_2/g_1.$$

Subtracting 1 from both sides of the equation and placing the gravity difference $(g_2 - g_1) = \Delta g$ obtain

$$\Delta g = g_1[R - 1]. \tag{7}$$

Here, as before, it is apparent that two observations suffice to obtain a relative gravity value. It is evident, also, that as many exchanges of the instruments between the two locations as desired may be made.

The second procedure described is the better of the two from the

standpoint of precision. In the result obtained by the first method the ratio of the S's enters as the second power, whereas in that obtained by the second method the ratio of the S's enters as the first power. Since the instruments furnish S's of equal precision it is evident that the result of the second procedure is exactly twice as precise as that of the first. This is illustrated, in a measure, by the probable errors of the results presented below.

DATA

The complete data for the two pairs of locations observed are exhibited in Tables 2 to 5 and are plotted in Figs. 2 and 3. Tables 2 and 3 and Fig. 2 present the data obtained when two pendulum instruments occupied the first pair of locations. Tables 4 and 5 and Fig. 3 present the data obtained when another two pendulum instruments observed the second pair of locations. Six observations, one hour in duration, were made in each case. The one hour interval is measured by a chronometer with sufficient accuracy to count the number of whole swings the pendulums make in that interval. The decimal part of the values appearing in the "Total Swings" (s) columns in Tables 2 and 4 is read from the photographic record made at the beginning and end of each observation for each instrument.

RESULTS—OBSERVATIONS ON THE FIRST PAIR OF LOCATIONS

Inspection of the plot of the S_2/S_1 and the S_2'/S_1' ratios in Fig. 2 shows that horizontal straight curves best fit these data. Hence, the value of the S_2/S_1 ratio may be determined as the average of observations 1, 2, and 6. This is

$$S_2/S_1 = 0.\ 998\ 401\ 02 \pm 3 \times 10^{-8}$$

where the probable error of 3×10^{-8} is subject to an uncertainty of from 30 to 50 per cent by reason of the small number of observations made.

A value for the S_2'/S_1' ratio may be determined, similarly, by averaging the ratios of observations 3, 4, and 5. This is

$$S_2'/S_1' = 0.\ 998\ 404\ 97 \pm 1 \times 10^{-8}$$

with a probable error with an uncertainty like the first.

Substituting these values in equation (4)

$$\Delta g = 980.1\left[(0.\ 998\ 404\ 97/0.\ 998\ 401\ 02)^2 - 1\right]$$
$$= 0.\ 007\ 76 \text{ gals.}^9$$

[9] The value for g_1 need be known roughly, only.

TABLE 2

PENDULUM INSTRUMENT WITH SUBSCRIPT (1)

Obs. No.	*Location*	*Total Swings*			$s_1/s_2=\mu$
1	on g_1	s_1	4065. 224	21	1. 000 001 25
		s_2	4065. 219	10	
2	on g_1	s_1	4065. 216	60	1. 000 001 10
		s_2	4065. 212	09	
3	on g_1	s_1	4065. 258	26	1. 000 001 23
		s_2	4065. 253	24	
4	on g_1	s_1	4065. 254	96	1. 000 001 28
		s_2	4065. 249	73	
5	on g_1	s_1	4065. 263	86	1. 000 001 13
		s_2	4065. 259	25	
6	on g_1	s_1	4065. 222	15	1. 000 001 09
		s_2	4065. 217	70	

PENDULUM INSTRUMENT WITH SUBSCRIPT (2)

Obs. No.	*Location*	*Total Swings*			$s_1/s_2=\mu$
1	on g_1	s_1	4058. 728	62	1. 000 003 41
		s_2	4058. 714	75	
2	on g_1	s_1	4058. 721	30	1. 000 003 66
		s_2	4058. 706	44	
3	on g_2	s_1	4058. 779	48	1. 000 003 97
		s_2	4058. 763	34	
4	on g_2	s_1	4058. 777	79	1. 000 004 73
		s_2	4058. 758	57	
5	on g_2	s_1	4058. 784	81	1. 000 003 66
		s_2	4058. 769	94	
6	on g_1	s_1	4058. 727	09	1. 000 003 61
		s_2	4058. 712	42	

The individual performance of the pendulums is best studied from the variation in the μ in the last column. The figure in the sixth decimal place indicates the precision with which the individual pendulums are adjusted in pairs. The variation in this figure, when it occurs, and in the figures following, indicates the amount by which horizontal seismic accelerations have affected the pendulum periods.

These ratios are plotted in Fig. 2.

If the precision measures for the two ratios are combined in a proper manner, a probable error for the result (subject to the uncertainty stated for each) is found. The value of Δg may be written

$$\Delta g = 7.76 \text{ milligals} \pm 0.06 \text{ milligals}.$$

TABLE 3

Obs. No.	Pend. Inst. with Sub. (1) S_1 and S_1'				Pend. Inst. with Sub. (2) S_2 and S_2'				Total Swing Ratios S_2/S_1 and S_2'/S_1'				
1	S_1	4065.	221	66	S_2	4058.	721	69	S_2/S_1	0.	998	401	08
2	S_1	4065.	214	35	S_2	4058.	713	87	S_2/S_1	0.	998	400	95
3	S_1'	4065.	255	75	S_2'	4058.	771	41	S_2'/S_1'	0.	998	404	94
4	S_1'	4065.	252	35	S_2'	4058.	768	18	S_2'/S_1'	0.	998	404	98
5	S_1'	4065.	261	56	S_2'	4058.	777	38	S_2'/S_1'	0.	998	404	98
6	S_1	4065.	219	93	S_2	4058.	719	76	S_2/S_1	0.	998	401	03

The columns headed S_1 and S_1' and S_2 and S_2' contain the total swing values for the average or "fictitious" pendulums and it is from these values that the effect of the horizontal seismic accelerations have been eliminated (though not the time duration of the runs). The total swing ratios S_2/S_1 and S_2'/S_1' are independent of both the length of time of the observations and the horizontal seismic accelerations. These comprise the working data from which the value of the gravity difference is obtained.

The total swing ratios are plotted in their proper time relationship to each other in Fig. 2.

The first method of field procedure was followed in the production of these data.

RESULTS—OBSERVATIONS ON THE SECOND PAIR OF LOCATIONS

Again, inspection of the plot of the S_2/S_1 ratios in Fig. 3 shows that horizontal straight curves best fit these data. Accordingly, the value of the ratio S_2/S_1 may be determined as the average of observations 1, 3, and 5. This is

$$S_2/S_1 = 1.\ 000\ 022\ 30 \pm 2 \times 10^{-8}$$

where the probable error of $\pm 2 \times 10^{-8}$ is subject to an uncertainty of from 30 to 50 per cent because of the small number of observations made.

The value of the S_2'/S_1' ratio may be determined in a like manner by averaging the ratios of observations 2, 4, and 6. This is

$$S_2'/S_1' = 0.\ 999\ 994\ 21 \pm 4 \times 10^{-8}$$

with a probable error with an uncertainty like the first.

Substituting these values in equation (7)

$$\Delta g = 980.1[(1.\ 000\ 022\ 30/0.\ 999\ 994\ 21) - 1]$$
$$= 0.\ 027\ 53 \text{ gals.}$$

If the precision measures for the two ratios are combined in a proper manner, a probable error for the result (subject to the uncertainty stated for each) is found. The value for Δg may be written

$$\Delta g = 27.53 \text{ milligals} \pm 0.05 \text{ milligals.}$$

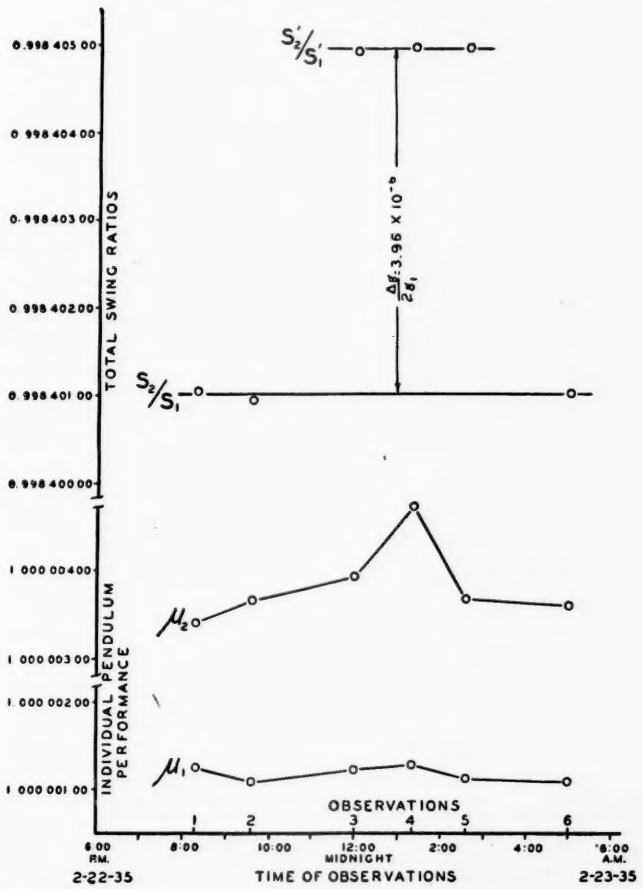

FIG. 2. Pendulum observations on the first pair of locations. The graph illustrates the extent to which the horizontal seismic accelerations are eliminated from the results due to the method of swinging two pendulums from a common support 180° apart in phase. The lower two curves are graphs of the μ ratios for the pendulums in the two cases used for these observations, μ_1 for instrument with subscript (1) and μ_2 for instrument with subscript (2). These curves exhibit, by their irregularity, the influence that the horizontal seismic accelerations have upon the periods of the individual pendulums. It may be noticed that the observations were taken at night when noise was a minimum and that the curve for instrument with subscript (1), μ_1, exhibits the least disturbance. This instrument was set on a very stable concrete pier during all six observations, whereas for observations 3, 4, and 5 instrument with subscript (2) occupied a field position. The operator's notes contain this comment opposite observation 4: "Train passed about $\frac{1}{4}$ mile away." This observation has the largest disturbance. However, when the average or fictitious pendulum values are used and their ratios plotted (the two upper curves), the irregularities or disturbances to the period are noticeably absent.

TABLE 4

PENDULUM INSTRUMENT WITH SUBSCRIPT (1)

Obs. No.	*Location*	*Total Swings*		$s_2/s_1=\mu$
1	on g_1	s_1	4037. 741 01*	1. 000 002 87
		s_2	4037. 752 62	
2	on g_2	s_1	4037. 818 97	1. 000 000 85
		s_2	4037. 822 42	
3	on g_1	s_1	4037. 755 22	1. 000 002 44
		s_2	4037. 765 08	
4	on g_2	s_1	4037. 818 75	1. 000 001 88
		s_2	4037. 826 37	
5	on g_1	s_1	4037. 766 27	1. 000 002 65
		s_2	4037. 777 01	
6	on g_2	s_1	4037. 809 23	1. 000 002 48
		s_2	4037. 819 28	

PENDULUM INSTRUMENT WITH SUBSCRIPT (2)

Obs. No.	*Location*	*Total Swings*		$s_1/s_2=\mu$
1	on g_2	s_1	4037. 841 41	1. 000 002 18
		s_2	4037. 832 59	
2	on g_1	s_1	4037. 803 58	1. 000 003 18
		s_2	4037. 790 73	
3	on g_2	s_1	4037. 853 19	1. 000 001 60
		s_2	4037. 846 70	
4	on g_1	s_1	4037. 803 27	1. 000 002 18
		s_2	4037. 794 44	
5	on g_2	s_1	4037. 865 92	1. 000 002 01
		s_2	4037. 857 78	
6	on g_1	s_1	4037. 795 39	1. 000 001 99
		s_2	4037. 787 35	

* The values in this column have been corrected for a small, constant rate of pressure increase occurring in the atmosphere within this instrument.

Individual pendulum performance can be studied from the variations in the μ. These ratios are plotted in Fig. 3.

It may be noted from the "Total Swings" column that the pendulums in the second instrument have periods that are in close adjustment with those in the first instrument.

THE SIGNIFICANCE OF THE PENDULUM OBSERVATIONS

The significant fact that is to be pointed out with regard to these measurements is that their precision is in no manner dependent upon

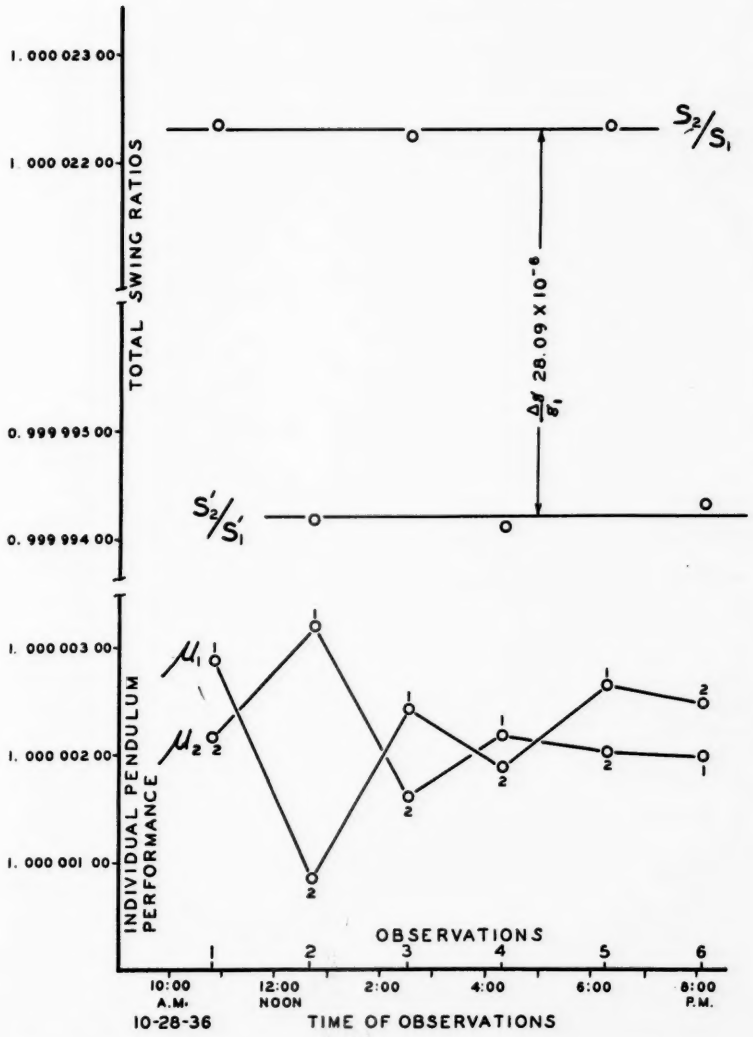

FIG. 3. Pendulum observations on the second pair of locations. The graph illustrates the elimination of horizontal seismic effects from the results of these observations in a manner even more striking than the graph of Fig. 2. The small figures 1 and 2 set beside the points of the μ_1 and μ_2 curves indicate that the instruments of those curves occupied stations where the acceleration of gravity had values g_1 and g_2 respectively. The sequence of these figures along the curves indicates the exchanges of locations by the two instruments.

TABLE 5

Obs. No.	Pend. Inst. with Sub. (1) S_1 and S_1'				Pend. Inst. with Sub. (2) S_2 and S_2'				Total Swing Ratios S_2/S_1 and S_2'/S_1'				
1	S_1	4037.	746	82*	S_2	4037.	837	00	S_2/S_1	1.	000	022	33**
2	S_1'	4037.	820	70	S_2'	4037.	797	16	S_2'/S_1'	0.	999	994	17
3	S_1	4037.	760	15	S_2	4037.	849	95	S_2/S_1	1.	000	022	24
4	S_1'	4037.	822	56	S_2'	4037.	798	86	S_2'/S_1'	0.	999	994	13
5	S_1	4037.	771	64	S_2	4037.	861	85	S_2/S_1	1.	000	022	34
6	S_1'	4037.	814	26	S_2'	4037.	791	37	S_2'/S_1'	0.	999	994	33

* See note appended to Table 4.

** The rate of the pressure increase was determined by previous observations extending over a period of six days. It amounted to 0.64×10^{-6} per day of 24 hours for the S_2/S_1 and S_2'/S_1' ratios. The pressure increased within the case. This tended to slow up the pendulums which made, consequently, fewer swings in the given time interval. Hence the ratio observed was too large. A correction proportional to elapsed time was applied to the data in this column.

The columns headed S_1 and S_1' and S_2 qnd S_2' contain the average or "fictitious" pendulum values.

The ratios column comprises the working data from which the gravity difference is obtained. These ratios are plotted in their proper time relationship to each other in Fig. 3.

The second method of field procedure was followed in the production of these data.

FIG. 4. One of the pendulum instrument set-ups for the second pair of locations mentioned in the article. The heat insulated pendulum case can be seen within the tent in front of the operator who is preparing to make the case level. The separate recording instrument extends from the case towards the end of the tent that is being held open by one of the two spectators.

the magnitude of the gravity difference measured. If it had been desired to measure a Δg of 100 milligals, instead of 25, the precision measures would not have changed; the precision is inherent in the method and with the pendulums.

ACKNOWLEDGMENTS

The author wishes to thank Dr. Paul D. Foote and Dr. E. A. Eckhardt of the Gulf Research & Development Company for permission to publish these data. He wishes also to express his sincere appreciation to those members of the laboratory staff who helped make the observations.

MEMORIAL

RAYMOND ANDREW LEMAY
1901–1940

Raymond Andrew LeMay, more widely known as "Pluto," one of the most colorful and most highly esteemed of the pioneers in geophysics, died suddenly of a heart attack at his home in Fort Worth, early on Monday, February 19, 1940. He had been in perfect health; the news came unexpectedly, and as a great shock to his family and friends.

He was born in Sherman, Texas, on October 30, 1901. The family not long after moved to Panama, where he first attended school. After six years, they returned to Fort Worth, where he won acclaim on the Central High School football team.

Continuing his athletic career in college, he was awarded the title of all-conference fullback at Daniel Baker College in Brownwood, in 1924. His success in football only served to call attention to his vivid, genial, dynamic personality. The people of Brownwood affectionately adopted him as a native son.

After obtaining his Bachelor's degree at Daniel Baker in 1925, he coached a season at Comanche High School, and then entered the Geophysical Department of the Gulf Oil Corporation (then the Gulf Production Company) at Houston, in January, 1926. He was one of the early members of our Society, joining in November, 1931.

As a field supervisor of a Gulf-Geophysical Research Corporation refraction party, he was in charge of the first American seismograph crew to discover a salt dome, that at Moss Bluff, Liberty County, Texas, in May, 1926. Three weeks later, his crew discovered the salt dome at Port Barre, Louisiana, and, about a year later, scored again at Bullard, Smith County, Texas. Later, he took two similar crews out to the Permian Basin, and played an important part in initiating the refraction program there which continued from 1928–1930. Later, as party chief of a reflection crew for Rosaire and Kannenstine, and, subsequently, for the Independent Exploration Company, his crews were, as they always had been, characterized by high morale, and field operations which were outstanding.

He was in charge of the Wichita Falls office of the Independent Exploration Company in 1938, and of its Tulsa office during 1939 and early 1940; acting at the same time, as assistant to the manager in the company's Fort Worth office.

He is survived by his widow, Iris Pauline LeMay; a daughter, Ray Allen LeMay; his mother, Mrs. E. A. LeMay; a sister, Miss Lumena LeMay; and three brothers, Paul, John, and Victor LeMay. All are of Fort Worth except John LeMay, who lives in Omaha.

"Pluto" LeMay displayed to best advantage the type of men who successfully introduced geophysical exploration, with no background of experience on which they could rely for guidance. LeMay was outstanding among his colleagues in his ability to meet and solve problems which never had arisen before, to recognize the importance of new phenomena, and to adapt himself to new and unforeseen conditions. At all times, he conducted himself as a man among men, earning the respect of his associates for his fearless integrity, his natural ability, and his strength and courage, physical as well as moral.

Although "Pluto" will be well remembered for his abilities, to the wide circle of friends whom he held close to him, his loss is deep and personal. His boyish good humor, his intense loyalty and the genuine kindness of his courtesies, are so vivid that he will ever live in the hearts of those who knew him.

E. E. ROSAIRE

March 1, 1940

DISCUSSIONS AND COMMUNICATIONS

NOTE ON THE CHANGE IN FREQUENCY OF THE REFLECTION FROM BASEMENT AS THIS REFLECTING HORIZON INCREASES IN DEPTH

Summary—A table is presented showing the variation in frequency of the reflection obtained from "basement" as this horizon increases in depth. The data were derived from seismograms obtained in the San Joaquin Valley, north of Bakersfield, California. The so-called "basement" reflecting horizon is followed from a depth of about 1300 feet on the east side to a depth of about 4400 feet farther out toward the middle of the Valley. The results indicate that the frequency of the reflection decreases by about 27 per cent while the depth of the reflecting horizon increases approximately 340 per cent. Gutenberg's equation, $T^2=T_0^2+aD$, where T is the period of the wave, D is the total distance travelled, and a is a constant, gives an approximate representation of these observations if $T_0^2=120$ and $a=0.05$ when T is expressed in thousandths of a second and D in feet.

It is generally known that the frequency of reflections obtained on seismograms is lower for the deeper than for the shallow reflections. On the assumption that this is due to the effect of internal friction Gutenberg[1] has shown that the period, T, is given as a function of the distance travelled, D, by the equation

$$T^2=T_0^2+aD$$

where T_0 is the period in the neighborhood of the origin, and a is a constant depending on the elastic constants and coefficient of internal friction of the material. It was thought that a comparison with this equation of data from seismograms obtained in routine field operations might be of some general interest, especially if the same reflecting horizon was used in all observations.

The table presented here was prepared from measurements taken on seismograms obtained in the usual manner in seismic prospecting in the San Joaquin Valley of California a few years ago. The change in depth of the reflecting horizon represents a variation of about 340 per cent of the original depth, while a corresponding decrease in frequency of about 27 per cent is observed. The depths tabulated are computed using a velocity-depth function which approximates the actual velocity in the area. The observed frequencies and periods were determined from the records by averaging all traces and the computed periods were derived from the above equation using $T_0=11$ milliseconds and $a=0.05$ (for T in milliseconds and D in feet) assuming $D=$ twice the computed depth. The reliability of the reflections used is indicated by the grade, G designating a superb reflection, F, a good reliable reflection, and P, an easily recognized and useable reflection. It will be noted that for a given depth of the reflecting horizon the variation of frequency with the charge of powder does not exceed 6 per cent. In most cases the difference between the observed and computed periods is of comparable order, and in no case is it greater than 10 per cent.

The rate of decrease of frequency with distance traveled, as indicated by these

[1] B. Gutenberg—"Handbuch der Geophysik," 4, 22 (1929).

Trace Time (Seconds)	*Grade*	*Computed Depth (Feet)*	*Charge (lbs.)*	*Frequency (~ per sec.)*	*Period (millisec.)*	*Computed Period (millisec.)*
0.440	*VP+*	1260	1	62.5	16.0	15.7
0.514	*F*	1500	2	56.3	17.8	16.5
0.662	*F*	1980	12	58.0	17.2	17.9
0.662	*F*	1980	6	61.5	16.3	17.9
0.722	*G*	2205	14	56.3	17.8	18.5
0.737	*P+*	2265	9	54.0	18.5	18.7
0.737	*P+*	2265	3	52.6	19.0	18.7
0.758	*G*	2310	9	54.0	18.5	18.8
0.758	*G*	2310	3	56.3	17.8	18.8
0.944	*G*	3000	3	50.0	20.0	20.5
1.020	*F+*	3245	6	45.5	22.0	21.1
1.124	*P*	3700	0.5	48.2	20.7	22.2
1.124	*P*	3700	9	50.0	20.0	22.2
1.124	*P*	3700	18	47.5	21.0	22.2
1.201	*P*	3965	1.5	42.5	23.5	22.8
1.298	*F*	4390	9	40.0	25.0	23.7
1.298	*F*	4390	1	40.8	24.5	23.7

figures, is somewhat greater than that found by Gutenberg[2] from refraction profiles in the Los Angeles Basin, his figures being $T=10$ milliseconds, $a=0.03$ in the units used above.

Due to the frequency discrimination of the recording system, the observed frequency is not necessarily the actual dominant frequency of the reflected wave. However, since the same discrimination was used on all records, it is believed that the rate of decrease indicated is of the correct order of magnitude.

SIDON HARRIS†
N. A. HASKELL‡

Jan. 26, 1940.

[2] B. Gutenberg, H. O. Wood, and J. P. Buwalda—Seismological Society of America Bulletin 22, 185 (1932).

† Stanolind Oil & Gas Co., Seismograph Dept.
‡ Western Geophysical Co. of California.

REVIEWS

Mathematical Methods in Engineering. Theodore v. Karman and Maurice A. Biot, McGraw-Hill Book Company (1940) 505 pp., $4.00.

The reviewer doesn't know if there are any mistakes in "Mathematical Methods in Engineering." In fact he didn't look for any. For when scientists of such high repute, as are v. Karman and Biot, write a book it seems rather petty to point out that on page x, line y a plus sign should have been minus.

Many books have been written in the last several years on engineering or applied mathematics. The present work is not new with respect to the general nature of the material it covers, though the individual personalities of the authors and their special fields of interest—aeronautics and mechanics—naturally are reflected in the selection of specific topics for treatment and in their choice of illustrations. It is, of course, very much up to date, and even up to the minute, as might be surmised from: "The calculation . . . gives . . . a good approximation for the motion of bombs dropped from aircraft." This book is, however, decidedly unique and apropos from the standpoint of manner of treatment. For, except for the first two chapters, it is not merely a book in advanced calculus profusely illustrated with mathematical exercises couched in physical phraseology. Rather it is a work on applied physics—especially mechanics and electric circuits—in which analytical procedures are developed primarily as tools to be used for the solution of physical and engineering problems. It is true that one will occasionally find the symbols Q.E.D.—so dreaded by students of high school geometry. But the careful reader cannot fail to become imbued with the satisfied feeling that after all mathematics can play the role of a servant just as well as that of a haunting terror for the engineer and physicist.

Chap. I gives a review of calculus and the theory of ordinary differential equations. The 23 pages of Chap. II are devoted to Bessel functions. Unfortunately here is one of the few places where the reviewer does not sympathize with the authors' point of view. While Bessel functions do appear in the solution of a great variety of physical problems the actual use of them in the present work is so limited that a special chapter for their discussion in a book of this kind hardly seems warranted.

"Fundamental Concepts of Dynamics," is the title of Chap. III. After a preliminary introduction on vector analysis a development is given of the basic laws of mechanics including the construction of Lagrange's equations. In Chap. IV, "Elementary Problems in Dynamics," particle motions under various laws of force are treated. The elements of elliptic integral theory are very nicely brought in so as to enable a more thorough understanding of the oscillation of a pendulum. Damping, resonance, ballistics, and airplane flight are also analyzed. The general theory of harmonic vibrations of conservative systems is developed in Chap. V, and a number of illustrations is happily included in which the analysis can be explicitly carried through for direct physical interpretation and discussion. And for the more complex systems where numerical work is necessary the reader is provided with a summary of not only the standard numerical methods of solving algebraic equations, but also the powerful method of matrices which has recently been getting increased attention from physicists and engineers. The small oscillations of dissipative systems is the subject matter of Chap. VI. The applications of the theory to wing flutter, airplane stability, and the vibration damper, are fully developed, as is the analogy between electrical and mechanical oscillations which is so frequently and profitably used throughout the rest

of the book. Another mathematical tool, the calculation of complex roots of algebraic equations, is also presented in this chapter.

A quite extensive treatment of the statics and dynamics of elastic structures together with interesting excursions into the general features of boundary value problems and the iterative procedure for the numerical determination of characteristic values is provided by Chap. VII. This subject is continued in Chap. VIII with special emphasis on the Fourier series method of attack. A brief résumé of practical harmonic analysis and an outline of the Rayleigh-Ritz variation method nicely show the practical possibilities of the Fourier series developments.

Chap. IX is entitled "Complex Representation of Periodic Phenomena." In it the concept of impedance, both electrical and mechanical, is developed and the complex representation of physical and analytical quantities is introduced. These are then used in Chap. X in the treatment of transient phenomena. After presenting the elements of Carson's theory, including the concept of the indicial admittance, the operator calculus is developed and illustrated.

The last chapter deals with the application of finite difference analysis to engineering problems. While only the most elementary aspects of the theory are developed, the numerous examples of application to elastic structures and to both mechanical and electrical filters clearly show the power of the theory.

A quasi-appendix entitled "Words and Phrases" follows the text. In this a number of exact definitions of mathematical terms is given. To the reviewer this seems to be of questionable value. It is doubtful if many engineers or physicists would be interested in these abstract definitions, and even those who might be would probably have difficulty in grasping the meaning of a new concept or term merely from a formal three to five line statement.

At the end of each chapter the authors have added not only valuable bibliographies of books for further study but a number of excellent problems of physical interest. The student will be interested to know that the answers to the problems are all listed at the end of the book.

Regardless of what set of mathematics or engineering books a student, teacher, or practising engineer may already have he will find this new work a most useful and valuable addition.

MORRIS MUSKAT

PATENTS

The following U. S. Patents have been issued since the listing in the January number. The list represents a major portion of those pertinent to the art and of interest to exploration geophysicists. The very brief abstracts are presented merely as an indication of the nature of the material covered by the patents so that the reader may judge whether further investigation is warranted. The abstracts are not to be construed in any sense as an interpretation of the claims.

ELECTRICAL PROSPECTING

U.S. No. 2,189,377, C. A. Heiland, Iss. 2/6/40 Appl. 2/23/37, Assign. Heiland Research Corp'n.

Method and Apparatus for Electrical Prospecting—Measuring higher order derivatives of surface potentials to determine depth of strata with improved resolution. Uses sets of three potential electrodes with the middle electrode of each set spaced from the nearest current electrode an integral number of times the spacing between potential electrodes. Uses bridge balance.

U.S. No. 2,190,320 Gennady Potapenko, Iss. 2/13/40 Appl. 12/22/37, Assign. Geo-Frequenta Corp'n.

Method of Determining the Presence of Oil—Measuring earth impedance due to polarization as a function of frequency between 1/100 and 100 cycles using earth electrodes.

U.S. No. 2,190,321, Gennady Potapenko, Iss. 2/13/40, Appl. 12/22/37, Assign. Geo-Frequenta Corp'n.

Method and Apparatus for Geophysical Prospecting—Earth is polarized at intervals. Between intervals impulses of opposite polarity are transmitted to test degree of polarization.

U.S. No. 2,190,322, Gennady Potapenko, Iss. 2/13/40, Appl. 12/24/37, Assign. Geo-Frequenta Corp'n.

Method of Geophysical Prospecting—Polarization vs. time is studied for build-up and decay. Uses repeated D.C. impulses and intermittent measurements.

U.S. No. 2,190,323, Gennady Potapenko, Iss. 2/13/40, Appl. 1/4/38, Assign. Geo-Frequenta Corp'n.

Method of and Apparatus for Geophysical Prospecting—Eearth is polarized for a period of time with D.C. The resultant polarization voltage is measured continuously during decay.

U.S. No. 2,190,324, Glen Peterson, Iss. 2/13/40, Appl. 2/2/38, Assign. Geo-Frequenta Corp'n.

Method of Geophysical Prospecting—Earth repeatedly polarized with D.C. and then measured for effect between periods of polarization. Time between polarizing and measuring periods may be varied. Length of polarizing periods or intensity of current may also be varied.

GEOCHEMICAL PROSPECTING

U.S. No. 2,183,964, Leo Horvitz, Iss. 12/19/39, Appl. 7/31/37, Assign. E. E. Rosaire.

Method of Exploration for Buried Deposits—The method of systematically taking samples of soil at or near the surface and analyzing the samples for hydrogen.

U.S. No. 2,192,525, E. E. Rosaire & Leo Horvitz, Iss. 3/5/40, Appl. 9/26/39, Assign. E. E. Rosaire.

Geophysical Prospecting Method—Detecting subterranean deposits from which leakage emanations occur by systematically collecting soil samples, confining samples from air contamination, removing said samples from confinement and analyzing the samples with respect to gases contained in the samples directly related to said deposits.

GRAVIMETRIC PROSPECTING

U.S. No. 2,185,582, L. W. Blau, Iss. 1/2/40, Appl. 8/29/35, Assign. Standard Oil Development Co.

Multiple Component Gravity Balance—Torsion balance has lower weight which may be suspended at two different elevations. Beam may rock in a vertical plane as well as rotate on the suspension in conventional manner. Records combining tilt and rotation are observed for 5 azimuths and with the lower mass at both elevations. This allows complete determination of the gravitational field including

$$\partial^2 u/\partial z^2,\ \partial^2 u/\partial y^2, \text{ and } \partial^2 u/\partial x^2$$

U.S. No. 2,190,959, Stephen Baron von Thyssen-Bornemisza, Iss. 2/20/40, Appl. 10/15/38, Assign. Shell Development Company.

Gravity Measuring Instrument—Astatized type gravimeter. Beam supported on knife edge with mass on one end and spring on the other. Astatization obtained by auxiliary mass rigidly mounted above the axis of beam or by an auxiliary mass pivotally supported so that for any motion of main mass auxiliary mass moves in such direction as to aid such motion.

GENERAL EQUIPMENT

U.S. No. 2,183,934, C. A. Heiland, Iss. 12/19/39, Appl. 6/4/37, Assign. Heiland Research Corp'n.

Multiple Galvanometer—Galvanometer elements with coils suspended by terminal strands. Shows half-turn coils with mirrors on non-conducting strips bridging the open sides of the coils.

SEISMOGRAPH PROSPECTING

U.S. No. 2,184,313, J. E. Owen, Iss. 12/26/39, Appl. 10/18/38, Assign. Geophysical Research Corp'n.

Seismic Surveying—Outputs of adjacent vertical and horizontal seismophones are adjusted in phase and amplitude and combined in opposition to cancel surface waves.

U.S. No. 2,184,953, A. B. Bryan, Iss. 12/26/39, Appl. 5/13/36, Assign. Standard Oil Development Co.

Time Lines Recording Apparatus—Timing lines produced by unsustained vibration of a tuning fork with shutter slits carried on the prongs. Vibration initiated electrically.

U.S. No. 2,192,973, A. I. Innes, Iss. 3/12/40, Appl. 11/4/37, Assign. Geophysical Research Corp'n.

Seismic Surveying—A method for selecting the most suitable depth of shot comprising use of conventional electrical-log in test shot hole, correlating best depth of shot in test hole with electrical-log and thereafter using electrical-log of subsequent shot holes for locating proper depth of shot.

U.S. No. 2,193,620, R. T. Cloud, Iss. 3/12/40, Appl. 9/20/37, Assign. Stanolind Oil & Gas Company.

Attenuator—Manual volume control for multi-channel seismograph comprising variable shunt and series resistors in each channel and ganged manual control of these attenuating units to achieve constant level recording.

U.S. No. 2,193,769, J. P. Minton, Iss. 3/12/40, Appl. 12/16/37, Assign. Socony-Vacuum Oil Company.

Frequency Multiplication System for Seismic Recording—Frequencies of seismic waves received are multiplied before passing through a filter to exclude low frequency ground-roll from record. Purpose of frequency multiplication is to facilitate filtering.

WELL LOGGING

U.S. No. 2,183,565, P. F. Hawley, Iss. 12/19/39, Appl. 5/27/38, Assign. Stanolind Oil & Gas Company.

Two-well Method of Electrical Logging and Apparatus Therefor—Current electrodes are placed in two separated wells and are lowered and raised individually to locate current maxima or minima to correlate strata in the two wells.

U.S. No. 2,184,338, G. H. Ennis, Iss. 12/26/39, Appl. 12/16/36, Assign. 1/2 R. V. Funk.

Method of Apparatus for Locating Water Leakages into Wells—Locating water entrance in wells by testing the regular well fluid electrically, allowing ingress of connate fluids and retesting. Fluid may be tested by measuring e.m.f. between electrodes of different metals spaced and centered in the well.

U.S. No. 2,190,686, L. B. Slichter, Iss. 2/20/40, Appl. 1/29/36, Assign. Schlumberger Well Surveying Corp'n.

Mineral Exploration—Combination of acoustic and electric or electromagnetic well logging. Cables and well instruments may serve combined acoustic and electric functions.

U.S. No. 2,191,119, Conrad Schlumberger, Iss. 2/20/40, Appl. 5/11/35, Assign. Schlumberger Well Surveying Corp'n.

Method and Apparatus for Surveying the Formations Traversed by a Bore Hole—Acoustic well logging. A local parameter such as intensity, velocity or damping is measured as an indication of lithographic character of formations traversed.

U.S. No. 2,191,120, L. B. Slichter, Iss. 2/20/40, Appl. 3/2/35, Assign. Schlumberger Well Surveying Corp'n.

Method of Geological Survey—Acoustic well logging method in which energy received at various points from a fixed acoustical source is compared.

U.S. No. 2,191,121, L. B. Slichter, Iss. 2/20/40, Appl. 1/5/40, Assign. Schlumberger Well Surveying Corp'n.

Geological Surveying Apparatus—Acoustic well logging comprising acoustic elements in streamlined case with spring or other contact member for coupling to wall of bore hole. Case may be split to facilitate clamping over a continuous cable.

THE SOCIETY ROUND TABLE

THE TENTH ANNUAL MEETING OF THE SOCIETY OF EXPLORATION GEOPHYSICISTS, CHICAGO, ILLINOIS, APRIL 9–11, 1940

The tenth annual meeting of the Society of Exploration Geophysicists was held in Chicago, Illinois, at the Stevens Hotel, April 9–11, 1940. The meeting was held concurrently with those of the American Association of Petroleum Geologists and the Society of Economic Paleontologists and Mineralogists. On Tuesday, April 9, and Thursday, April 11, sessions were held in the Upper Tower Ballroom, with registration in the Lower Tower Ballroom, while a joint meeting with the other two associations was held Wednesday, April 10, in the Grand Ballroom.

Although all who attended the sessions did not register, there was a registration of 206. This number included 123 active members, 9 associate members, and 74 non-members. In addition, 30 wives were noted as being in attendance.

Mr. W. T. Born arranged the excellent program of papers, and arrangements are being made to send each member not in attendance a copy of the program. Papers listed in the program will be published in GEOPHYSICS during the coming year.

MINUTES OF THE ANNUAL BUSINESS MEETING OF THE SOCIETY OF EXPLORATION GEOPHYSICISTS HELD IN THE UPPER TOWER BALLROOM OF THE STEVENS HOTEL, CHICAGO, ILLINOIS, APRIL 9, 1940

The meeting was called to order by the President, E. A. Eckhardt, in the Upper Tower Ballroom of the Stevens Hotel, Chicago, Illinois, at 10:00 A.M., April 9, 1940. The minutes of the previous business meeting were then read by the Secretary-Treasurer, J. H. Crowell. Upon motion being duly made and seconded and put to a vote, the minutes were approved.

The Secretary-Treasurer then made a brief financial report based on the annual audited statement for the year ending December 31, 1939. The Society showed an increase in assets of $3,073.32 over 1938. The membership of the Society was given as 821.

Vice-President W. T. Born submitted to the Society the recommendation that certain revisions in Article V-A-2 of the Constitution be made as outlined by the Business Committee. (These amendments will be noted on the ballot which will soon be mailed members.) A motion to adopt the revisions was duly made and seconded and upon being put to a vote, there being no discussion, was unanimously approved.

The results of the balloting were then announced as follows:

President, W. T. Born
Vice-President, H. B. Peacock
Editor, R. D. Wyckoff
Secretary-Treasurer, Andrew Gilmour

There being no further business, a motion for adjournment was duly made and seconded. Upon being put to a vote, the motion was passed without dissent.

Respectfully submitted,

JOHN H. CROWELL, *Secretary-Treasurer*

MINUTES OF THE SECOND BUSINESS MEETING OF THE SOCIETY OF EXPLORATION GEOPHYSICISTS HELD IN THE UPPER TOWER BALLROOM OF THE STEVENS HOTEL, CHICAGO, ILLINOIS, APRIL 11, 1940

The meeting was called to order by President W. T. Born in the Upper Tower Ballroom of the Stevens Hotel, Chicago, Illinois, at 2:30 P.M., April 11, 1940. Minutes of the previous meeting were not read because of the lack of time.

The recommendation of the business committee to the effect that the Constitution be amended to provide a third class of membership for inexperienced students sponsored by their department heads was put before the meeting. It was moved by R. F. Beers and seconded by W. M. Rust, Jr., that this amendment (to be balloted soon by active members) be adopted. Upon being put to a vote, it was passed unanimously.

Announcement was made of the election of Donald C. Barton to posthumous Honorary Membership in recognition of his work in founding the Society and in furthering the science of geophysics.

There being no further business, upon motion being duly made, seconded, and put to a vote, the meeting was adjourned.

Respectfully submitted,
JOHN H. CROWELL, *Secretary-Treasurer*

ANNUAL REPORT OF THE SECRETARY-TREASURER OF THE SOCIETY OF EXPLORATION GEOPHYSICISTS

The financial condition of the Society of Exploration Geophysicists as of March 31, 1940, is shown by the following balance sheet:

ASSETS

Cash in Banks			
Austin National Bank (checking account)	$4,069.86		
Austin National Bank (saving account)	2,006.66	$6,076.52	
Guardian Trust Company (checking account)	611.06		
Guardian Trust Company (savings account)	2,042.79	2,653.85	
National Bank of Tulsa (savings account)		1,535.87	$10,266.24
Accounts Receivable			
Advertising		261.20	
Back Numbers		39.30	
Dues—1939		360.00	
Dues—1940		1,492.23	
Reprints		352.80	
Subscriptions		37.45	2,542.98
Office Equipment			
File Cabinet	$19.50		
Sectional Bookcase	12.00	31.50	
Less: Reserve for Depreciation		4.65*	26.85
			$12,836.07

LIABILITIES

Accounts Payable		
General		101.70*
Surplus		
Excess of Income over Expense December 31, 1939	$8,530.01	
Excess of Income over Expense 1940 to Date	4,407.76	$12,937.77
		$12,836.07

The operating gain of $4,407.76 for 1940 to date is detailed as follows:

Income		
Advertising	$ 622.50	
Back Numbers	303.80	
Dues	4,138.00	
Reprints	68.88	
Subscriptions	773.30	$ 5,906.48
Expense		
Advertising Commissions	55.37	
Auditing	104.50	
Bank Charges	1.42	
Bond Premiums	10.00	
Express and Freight	.35	
Miscellaneous	21.00	
Postage	81.57	
Publications	799.32	
Reprints	59.89	
Stationery and Office Supplies	60.65	
Stenographic and Clerical Services	300.00	
Telephone and Telegraph	4.65	1,498.72
To Balance Sheet		$ 4,407.76

Hereafter the financial report presented to the Society will be that covering the preceding year and will thus represent the annual audit. Balance sheets prepared just prior to the annual meetings are erroneous in that they show the bulk of the current year's income, but little of the expense. The balance sheet prepared for the 1939 audit is reproduced below.

BALANCE SHEET—DECEMBER 31, 1939

ASSETS

Current			
Cash			
In Banks		$7,607.10	
Accounts Receivable			
Membership Dues	$460.50		
Reprints	427.45		
Advertising	312.60		
Subscriptions	25.40		
Back Numbers	35.28		
Advances	100.00	1,361.23	$8,968.33

Office Equipment		
Steel File and Book case	31.50	
Less: Allowance for Depreciation	4.65	26.85
		$8,995.18

LIABILITIES

Current		
Accounts Payable		
For Expenses, etc.		$ 214.50
Deferred Income		
Prepaid Advertising, Dues, and Subscriptions		250.67
Surplus		
Balance January 1, 1939	5,597.66	
Add		
Increase from Current Year Operations—Per Statement of Income and Expense	2,932.35	8,530.01
		$8,995.18

INCOME AND EXPENSE—DECEMBER 31, 1939

INCOME

Membership Dues			
Dues	$4,052.50		
Reinstatements	36.00	$4,088.50	
Publications			
Advertising	2,650.70		
Subscriptions	1,057.85		
Back Numbers	651.52		
Reprints	622.66	4,982.73	
Other Income			
Interested Received		60.89	
		$9,132.12	
Less: Discounts to Colleges, etc.			
On Subscriptions and Back Numbers		108.43	$9,023.69

COSTS AND EXPENSES

Publication Costs			
Geophysics:			
Vol. IV, No. 1—104 Pages	$ 645.18		
No. 2— 98 Pages	530.53		
No. 3—114 Pages	662.14		
No. 4—146 Pages	813.96		
	$2,651.81		
Reprints	624.33	$3,276.14	

Expenses				
Salary-Business Manager		$1,200.00		
Commissions:				
Advertising:				
Business Manager	$287.67			
Agencies	257.73			
Subscriptions and Back Numbers:				
Agencies	38.50	583.90		
Postage and P.O. Box Rent		229.26		
Stationery and Office Supplies		182.05		
Traveling Expense		169.42		
Bad Debts		134.50		
Meeting Expense:				
Programs	61.00			
Radio Loud Speaker	20.00			
Machine Screen Operator	15.00			
Badges and Show Cards	13.39	109.39		
Auditing		100.00		
Flowers		25.00		
Telephone and Telegraph		22.55		
Society Card, etc.—A.A.P.G.		18.00		
Bond Premiums		15.00		
Express and Freight		14.58		
Repairs Typewriter		4.25		
Bank Charges		3.65		
Depreciation		3.15		
Notary Fee		.50	2,815.20	6,091.34
Excess of Income Over Costs and Expenses				$2,932.35

As of January 1, the Society had on hand approximately 4,600 copies of GEOPHYSICS with a minimum value, at $2.00 each, of $9,200. This is in addition to the amount indicated on the balance sheet.

The increase in membership is shown in the following figures:

Year	*Honorary*	*Active*	*Associate*	*Total*	*Increase*
1930	0	34	5	39	
1931	2	68	11	81	42
1932	2	83	11	96	15
1933	2	91	10	103	7
1934	2	141	23	166	63
1935	2	156	31	189	23
1936	4	195	34	233	44
1937	4	367	112	483	250
1938	4	510	158	672	189
1939	4	632	185	821	149

Respectfully submitted,

JOHN H. CROWELL, *Secretary-Treasurer*

PERSONAL ITEMS

(Members are requested to send material for publication herein to J. F. Gallie, P. O. Box 777. Austin, Texas.)

H. W. Straley, III, of Baylor University, has been appointed Secretary of the Geophysical Education Committee of the Educational Division of the American Institute of Mining and Metallurgical Engineers.

Keith H. Odenweller has been transferred from Trinidad to South America. He may be addressed at Apartado 78, Ciudad Bolivar, Venezuela.

W. C. Merritt has been transferred from Suriman to South America. He may be addressed at Apartado 1329, Caracas, Venezuela.

John W. Flude has begun production of the plastic shot-hole casing developed by him during the last several years. He has organized the Southern Plastic Company, located at 1500 Heiner St., Houston, Texas, to manufacture the light-weight Fedralite casing.

S. Zimerman has been transferred from Jackson to Meridian, Mississippi. He may be addressed at Box 391.

Merle C. Bowsky of the Lane-Wells Company has been transferred from Los Angeles, California, to Houston, Texas. His home address in that city is 1841 Richmond St.

W. G. McGuire of the Magnolia Petroleum Company was killed in an automobile accident January 2nd, Paul E. Nash has advised the Society. Mr. McGuire became an associate member of the Society in 1938.

Philip M. Konkel has left the General Geophysical Company to accept a position as geologist with the Ohio Oil Company in Marshall, Illinois.

Sidon Harris of the Stanolind Oil and Gas Company has moved his office from Amarillo to Fort Worth, Texas. The company's address in that city is Box 1410.

E. E. Rosaire has advised that offices and laboratory of Subterrex have been moved to 701 Waugh Drive, Houston, Texas.

George B. Lamb of the Gulf Research & Development Company has been transferred from Pittsburgh, Pennsylvania, to Houston, Texas. His mailing address in Houston is P. O. Drawer 2100.

John A. Pranglin, formerly with the Mott-Smith Corporation in Egypt, has accepted a position with the Lane-Wells Company, P. O. Box 389, Corpus Christi, Texas.

George M. Bevier has moved to San Antonio, Texas, to do independent geophysical and geological work. His office is located at 911 Milam Building.

E. L. DeGolyer has been awarded the Anthony F. Lucas gold medal by the American Institute of Mining and Metallurgical Engineers for distinguished achievement in improving the technique and practice of finding and producing petroleum.

W. B. Agocs of the Gulf Research and Development Company may be addressed at P. O. Drawer 2038, Pittsburgh, Pennsylvania.

The United Geophysical Company has opened a new laboratory at 1255 East Green Street, Pasadena, California. The new building will provide much better quarters for the conduct of seismograph operations and for the company's various research projects, President Herbert Hoover, Jr., advises.

Charles C. Lash of the Western Geophysical Company is now located in Hanford, California, at 330 Porter St.

George C. McGhee has left the National Geophysical Company to become a member of the firm of DeGolyer, MacNaughton and McGhee, with offices in the Continental Building, Dallas, Texas.

The Seismograph Service Corporation has opened a division office at 628 Lampton Building, Jackson, Mississippi. Stanley W. Wilcox is dividing his time between this office and that in Flora, Illinois.

Edward S. Foster, Jr., formerly with Subterrex, has accepted a position with the Halliburton Oil Well Cementing Company. His home address in Houston is 603 Edgewood St.

Homer E. Roberts has been transferred from Karachi, India, to Rangoon. Mail may be addressed to Box 760, Rangoon, Burma.

G. W. Postma has been transferred from Balik, Papan to Sangasanga, Borneo. He may be addressed c/o B. P. M. Sangasanga, Borneo, N. E. I.

C. A. Heiland of the Heiland Research Corporation and the Colorado School of Mines discussed "Soil and Gas Analysis Methods of Prospecting" in Denver, Colorado, in January. The paper was presented before the Rocky Mountain Association of Petroleum Geologists.

P. M. Schoorel has been transferred from South America to Tarakan, Borneo, Dutch East Indies, where he will work for De Bataafsche Petroleum Mij.

Charles A. Durham of the Humble Oil and Refining Company has been transferred from Kingsville, Texas, to Houston. He may be addressed at 850 Humble Building.

J. C. Karcher, president of the Coronado Corporation of Dallas, Texas, has been elected a director of the Republic National Bank of that city.

Herbert Hoover, Jr., speaking before the A.I.M.M.E. in New York in February, discussed the work done by his company, United Geophysical, in the detection of soil gases with the mass spectrograph.

Denys H. Back, formerly with S. A. Judson, is vice-president of the Electro Geophysical Exploration Company, with offices in the First National Bank Building, Houston, Texas.

Nat H. Prade has accepted a position in the geophysical laboratory of the Sun Oil Company in Beaumont, Texas.

F. A. Roberts has been transferred from Babo, Netherlands New Guinea, to Soengei Gerong, Sumatra, N. E. I. He may be addressed c/o N. V. N. K. P. M.

E. L. DeGolyer of DeGolyer, MacNaughton and McGhee, Dallas, and distinguished professor at the University of Texas, Austin, has been elected a director of the First National Bank of Dallas, Texas.

Robert Bliss has returned to his home in Greeley, Colorado. He may be addressed at Route 1, Box 9, in that city, where the death of his father has necessitated his remaining.

H. C. Becker has left the Barnsdall Oil Company to return to a position with the Seismograph Service Corporation, 709 Kennedy Building, Tulsa, Oklahoma.

Elmer W. Ellsworth, formerly with W. C. McBride, Inc., is now serving as a geologist for the Independent Oil Producers Association of Illinois, with offices in the Wham Building, Centralia, Illinois.

F. L. Bishop is now serving as District Representative of the Mid-Continent Area for Geophysical Service, Inc., with offices at 1613 National Bank of Tulsa Building, Tulsa, Oklahoma.

G. H. Westby, president of the Seismograph Service Corporation, discussed "Well Logging by Radioactivity" at Tulsa, March 11, before the Mid-Continent Section of the A.I.M.E.

G. L. Barksdale of the Petty Geophysical Engineering Company has been transferred from Venezuela to Apartado 100, Cucuta, Colombia, South America.

James A. Moore is now employed by the Union Sulphur Company in Sulphur, Louisiana.

Henry R. Kimble of the Magnolia Petroleum Company has been transferred to South America, where he may be addressed c/o Socony-Vacuum Oil Company, Apartado 246, Caracas, Venezuela.

Gas Laboratories, Inc., has opened an office in the Commerce Building, Houston, Texas. E. O. Buck and A. S. Parks, consulting engineers and geologists, organized the company.

Louis J. Paddison has been transferred from Dodge City, Kansas, to Fort Worth, Texas. He may be addressed c/o Stanolind Oil and Gas Company, Box 113, Sylvania Station.

Lyman Chandler, Jr., is now with Yacimientos Petroliferos Fiscales, Avda. R. Saenz Pena 777, Buenos Aires, Argentina.

H. M. Horton has resigned his position with the Superior Oil Company to become an independent operator. He is now located in Evansville, Indiana, at 312 Claremont Apartment Hotel.

Malvin G. Hoffman has resigned his position with the Midco Oil Corporation of Tulsa, and is now an independent geologist. He may be addressed at 1321 South Quaker Avenue, Tulsa, Oklahoma.

Elliott Sweet has moved from New York to Apartment 206, 1924 North Argyle Street, Hollywood, California.

W. B. Perry is returning to the States from Trinidad. He may be addressed at 120 North Philadelphia, Shawnee, Oklahoma.